Extended Graetz Problem Including Axial Conduction and Viscous Dissipation in Microtube

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Extended Graetz problem in microtube is analyzed by using eigenfunction expansion to solve the energy equation. For the eigenvalue problem we applied the shooting method and Galerkin method. The hydrodynamically isothermal developed flow is assumed to enter the microtube with uniform temperature or uniform heat flux boundary condition. The effects of velocity and temperature jump boundary condition on the microtube wall, axial conduction and viscous dissipation are included. From the temperature field obtained, the local Nusselt number distributions on the tube wall are obtained as the dimensionless parameters (Peclet number, Knudsen number, Brinkman number) vary. The fully developed Nusselt number for each boundary condition is obtained also in terms of these parameters.

**Key Words**: Graetz Problem, Microtube, Slip Boundary Condition, Viscous Dissipation, Axial Conduction, Eigenvalue Problem, Knudsen Number, Peclet Number, Brinkman Number

**Nomenclature**

\[ A_n : \text{Coefficients} \]
\[ Br : \text{Brinkman number} \]
\[ Br = \frac{\mu \omega_m^2}{k(T_0 - T_w)} \]
\[ C_1 : 1 + 8 \frac{2 - F}{F} K_n \]
\[ C_2 : \frac{2 - F_t}{F_t} \frac{2 \gamma}{\gamma + 1} \frac{K_n}{Pr} \]
\[ c_p : \text{Specific heat} \]
\[ D : \text{Diameter of microtube} \]
\[ F : \text{Tangential momentum accommodation coefficient} \]
\[ F_t : \text{Thermal accommodation coefficient} \]
\[ h : \text{Heat transfer coefficient} \]
\[ k : \text{Thermal conductivity} \]
\[ K_n : \text{Knudsen number} \]
\[ L : \text{Length of microtube} \]

\[ Nu : \text{Nusselt number} \quad (Nu = hD/k) \]
\[ \rho : \text{Pressure} \]
\[ Pe : \text{Peclet number} \quad (Pe = Re \cdot Pr = w_m D / \alpha) \]
\[ Pr : \text{Prandtl number} \quad (Pr = \nu / \alpha) \]
\[ q : \text{Heat flux} \]
\[ R : \text{Radius of microtube} \]
\[ R_n : \text{Eigenfunction} \]
\[ r, z : \text{Cylindrical coordinates} \]
\[ Re : \text{Reynolds number} \quad (Re = w_m D / \nu) \]
\[ T : \text{Temperature} \]
\[ w : \text{Fluid velocity} \]

**Greek symbols**

\[ \alpha : \text{Thermal diffusivity} \]
\[ \beta : \text{Eigenvalue} \]
\[ \gamma : \text{Specific heat ratio} \]
\[ \lambda : \text{Molecular mean free path} \]
\[ \mu : \text{Dynamic viscosity} \]
\[ \nu : \text{Kinematic viscosity} \]
\[ \theta : \text{Dimensionless temperature} \]
\[ \theta = \frac{(T - T_w)}{(T_0 - T_w)} \quad \frac{k(T - T_0)}{(q_w R)} \]

**Subscript**

\[ m : \text{Mean values} \]

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s : Fluid properties at the wall  
w : Wall values  
0 : Inlet properties  
∞ : Infinite properties

Superscript
* : Dimensionless variables

1. Introduction

The recent development of microfabrication technologies such as deep X-ray lithography and silicon-based micromachining has made it possible to design of microfluidic devices with microscale dimensions. Microfluidic systems for manipulating fluids in the microscale are widely used in the application areas such as chemistry, biology, material science and MEMS etc. For example, to design the cooling system of electronic devices as micro heat exchangers, the knowledge of convection heat transfer in microscale cylindrical or rectangular passages is required. Many investigations have been performed during the last two decades for convection heat transfer in microsystems and some of the experiments have shown that fluid flow and heat transfer characteristics in microgeometry deviate from the well known traditional approaches based on the continuum assumption (Tuckerman and Pease, 1981 ; Choi et al., 1991).

For the flow in microtube, the no-slip boundary conditions need to be modified as the radius of the tube is reduced and slip velocity and temperature jump may occur on the wall. The slip boundary condition may be used when gases are at low pressure or for flow in extremely small passages. The rarefaction effects of a gas are included from the Knudsen number $Kn$, the ratio of the mean free path of the gas to the characteristic length of the flow field. Karniadakis and Beskok (2002) have proposed the range of the Knudsen number for slip flow as $0.001 < Kn < 0.1$.

The Graetz problem is a simplified problem of forced convection heat transfer in a circular tube in laminar flow, which was solved by Graetz (1883 ; 1885) analytically assuming fully developed laminar flow and neglecting axial conduction and viscous dissipation. Sellars et al. (1956) extended the Graetz problem using a more effective approximation technique for evaluation of the eigenvalues problem. Lahjomri and Oubarra (1999) solved the problem to include the effect of axial conduction in Graetz problem. Barron et al. (1997) and Ameel et al. (1997) presented an analytic solution including slip effect for uniform temperature and uniform heat flux boundary conditions on the circular tube, respectively. Tunc and Bayazitoglu (2001) solved the energy equation with slip velocity and temperature jump boundary conditions in a microtube, including viscous dissipation but neglecting the axial conduction. In the most analysis of the Graetz problems extended, the both effects of axial conduction and viscous dissipation are not included. But, both axial conduction and viscous dissipation may not be ignored, if liquid metal is working fluid and fluid velocity is high.

In this paper, we consider the extended Graetz problem in the circular microtube including the effects of rarefaction, axial conduction and viscous dissipation altogether. At the entrance, the temperature starts to be developed from uniform temperature while the flow is assumed to be fully developed Poiseuille flow. Two types of heat boundary condition on the wall, isothermal and constant heat flux, are considered. By using the eigenfunction expansion method, the temperature distributions in the microtube are determined, and Nusselt number distributions on the wall are shown for some typical values of the parameters (Knudsen number $Kn$, Peclet number $Pe$, and Brinkman number $Br$). Nusselt number at far downstream of the tube is obtained as a function of the parameters.

2. Analysis

2.1 Uniform temperature on the wall

The steady-state hydrodynamically developed flow with constant temperature $T_0$ enters into the microtube as illustrated in Fig. 1. The fluid temperature would change from the value $T_0$ at the entrance to the value $T_w$ on the walls. Assuming