A Finite Thin Circular Beam Element for In–Plane Vibration Analysis of Curved Beams

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In this paper, the stiffness and the mass matrices for the in–plane motion of a thin circular beam element are derived respectively from the strain energy and the kinetic energy by using the natural shape functions of the exact in–plane displacements which are obtained from an integration of the differential equations of a thin circular beam element in static equilibrium. The matrices are formulated in the local polar coordinate system and in the global Cartesian coordinate system with the effects of shear deformation and rotary inertia. Some numerical examples are performed to verify the element formulation and its analysis capability. The comparison of the FEM results with the theoretical ones shows that the element can describe quite efficiently and accurately the in–plane motion of thin circular beams. The stiffness and the mass matrices with respect to the coefficient vector of shape functions are presented in appendix to be utilized directly in applications without any numerical integration for their formulation.

Key Words: Thin Circular Beam, Finite Element, In–plane Motion, Natural Shape Function, Stiffness Matrix, Mass Matrix, Shear Deformation, Rotary Inertia

1. Introduction

The in–plane static or vibration analysis of curved beams is quite complex due to the presence of bending–extension coupling and the effects of shear deformation and rotary inertia. Neglecting these effects may lead to inaccuracies of the analysis especially when the ratio of radial thickness to radius of curvature of a curved beam is not small (thick circular beam), and for vibration problem, natural frequencies of higher modes may be erroneous even though the ratio is very small (thin circular beam).

So far, many papers studying the finite curved beam elements for the in–plane static or vibration analysis of non–straight beams have been reported. The total extent of the works in this field is now too great to be reviewed in detail, but those papers that set the present work in context are briefly summarized in the following.

Davis et al. (1972) presented the shape functions of a curved beam element that are the exact in–plane displacements obtained from an integration of the differential equations of an infinitesimal element in static equilibrium. The stiffness and mass matrices were derived from the force–displacement relations and the kinetic energy equations, respectively. The matrices are formed for the in–plane motion of either a thick curved beam with the effects of shear deformation and rotary inertia or a thin curved beam without those effects. The matrices are formulated in the local straight–beam (Cartesian) coordinate system rather than in the local curvilinear (polar) coordinate system, and thus a transformation of the matrices for the local coordinate system to the one for the common global coordinate system is required before its are assembled even though the radius of curvature for the entire curved beam is

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constant.

Ashwell et al. (1971) studied the static application of cylindrical shell element and compared the results with those obtained by using a new element based on simple strain function. A finite element formulation based on natural shape function was also presented but the effect of shear deformation was not considered. Yamada and Ezawa (1977) proposed a straightforward criterion to warrant the displacement functions being used in the finite element approximation of circular arches. The criterion was established by studying the natural shape functions of the exact solutions of the deformed shape of the circular arch element. Meck (1980) developed a finite element solution for a thin curved beam by considering or neglecting the extensional deformation and observed that excellent results could be obtained by using polynomial based displacement functions.

Prathap and Babu (1986) derived a 3-node curved beam element with shear deformation, based on independent iso-parametric interpolations and field consistency principles. This beam element suffered from membrane locking with the increase in the ratio of element length to thickness due to the inconsistency of membrane strain, while the inconsistency in shear strain did not lead shear locking, but degraded the performance of the element and resulted in severe force oscillations. Guimaraes and Heppler (1992) investigated a thin beam element based on trigonometric functions for its ability to recover incremental rigid body motions. They compared the performance of three different models. Choi and Lim (1993) developed two curved beam elements, which are the CSCC and the CSLC elements based on Timoshenko beam theory and curvilinear coordinate system, modified from the conventional strain based shape function element. Lee and Sin (1994) presented the formulation of a 3-node curved beam element based on curvature.

Sabir et al. (1994) developed a strain based curved beam element by using Timoshenko deep beam formulation in the curvilinear coordinate system. A linear variation of bending curvature, and a constant extensional as well as shear strains were chosen. Krishnan and Suresh (1998) obtained the effects of shear deformation on deflection and shear deformation together with rotary inertia on the natural frequencies of curved beams by using a 2-node cubic–linear curved beam element having 4 degrees of freedom per node in local Cartesian coordinate system. Kim and Kang (2003) presented a new highly accurate two-dimensional curved composite beam element based on the Hellinger–Reissner variational principle and classical lamination theory by employing consistent stress parameters corresponding to cubic displacement polynomials with additional nodeless degrees.

In the past four decades, some novel approaches for curved beam elements have been presented, but they are not widely adopted in the practical applications because of their complexity. To avoid the complex formulations by the existing approaches, this paper is concerned about development of a thin circular beam element, i.e. a curved beam element of which radial thickness is very small as compared with radius of curvature and in which the effect of variation in curvature across the cross section is neglected. In this paper, the stiffness matrix and mass matrix for the in-plane motions of a thin circular beam element are derived respectively from the strain energy and the kinetic energy, in different manner of the works by Davis et al. (1972), by using the natural shape functions of the exact in-plane displacements which are obtained from an integration of the differential equations of a thin circular beam element in static equilibrium. The matrices are formulated in the local polar coordinate system with the effects of shear deformation and rotary inertia. If necessary, the matrices can be transformed without difficulties for the global Cartesian coordinate system. Some numerical examples are performed to verify the element formulation and its analysis capability. The results obtained by using FEM are compared with the theoretical ones to examine the convergence and accuracy of the element. The stiffness and the mass matrices with respect to the coefficient vector of shape functions are presented in appendix.