Chaotic synchronization based on nonlinear state-observer and its application in secure communication

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Abstract: Chaotic synchronization is a branch of chaotic control. Nowadays, the research and application of chaotic synchronization have become a hot topic and one of the development directions is for the research on chaos. In this paper, a universal nonlinear state-observer is presented for a class of universal chaotic systems to realize the chaotic synchronization, according to the theory of state-observer in the modern control theory. And theoretic analysis and simulation results have illustrated the validity of the approach. Moreover, the approach of synchronization proposed in this paper is very easy, flexible and universal with high synchronization precision. When the approach is applied to secure communication, the results are satisfying.

Keywords: chaotic synchronization; nonlinear state-observer; synchronization error; secure communication

0 INTRODUCTION

In recent years, the research on chaotic synchronization has been paid more and more attention to, and chaotic synchronization has become a key problem in chaotic secure communication. Chaotic synchronization refers to a process wherein two chaotic systems (either equivalent or nonequivalent) adjust a given property of their motion to a common behavior due to a coupling or to a forcing (periodical or noisy). At present, there are many ways for chaotic synchronization, not only traditional feedback, but also sliding-mode and adaptive synchronization etc. Ref. [1] makes use of a way of synchronization control to realize the synchronization of state variables between two different chaotic systems, but all the variables must be got and joined into control. In fact, not all the states of a general nonlinear system can be got, so the theory of observer is imported to the research of chaos synchronization. Now, the research on chaotic synchronization based on state-observer have got some results, Ref. [2] only needs to transmit a scalar signal to synchronize a special hyper-chaotic system without calculating the exponent of Lyapunov and the restriction of attraction fields. But it can only dispose of the hyper-chaotic system with one nonlinear item. Ref. [3] realizes the synchronization of a type of chaotic system, combining state-observer with phase space together, but the controller designed is too complex. A nonlinear state-observer with a controller for a type of chaotic system in which not all states can be measured is presented in Ref. [4], but the model of chaotic system is not universal.

As we all know, it is difficult to design nonlinear observer, the research on observer was only for a given chaotic system before. All kinds of ways[1-4,6-11] of chaotic synchronization are only for some certain systems or similar systems in model, which can be unified to a universal nonlinear system model. To this day, there isn’t a systematic approach for a universal chaotic system. In this paper, a universal nonlinear state-observer with a nonlinear controller is presented for a universal nonlinear chaotic system to realize the chaotic synchronization, according to the state-observer theory of modern control theory. And the synchronization system is robust strongly. Moreover, the approach is easy, alive and universal, and can accomplish some chaotic synchronization which can’t be realized by ancient ways, even when the nonlinear item of chaotic systems observed includes some uncertain ingredients (uncertain parameters, etc).

1 CHAOTIC SYNCHRONIZATION SYSTEM

Generally, considering a nonlinear system of the form

\[ X = AX + BF(X) + C, X(0) = X_0, \]  

\[ Y = KX + F(X), \]
where $A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{m \times n}, C \in \mathbb{R}^n, F : \mathbb{R}^m \rightarrow \mathbb{R}^n (m \leq n)$ is a nonlinear mapping, $AX$ is the part of linear, $BF(X)$ is the part of nonlinear, $C$ is a constant matrix. $Y$ is the output of the system, $K \in \mathbb{R}^{n \times n}$ is the coefficient matrix. And formula (1) is the aim system, formula (2) is the output system, where $A$ and $K$ satisfy the condition of observability.

The application value of chaotic synchronization theory is to realize the copy and reappearance of the aim system. The traditional state-observer copies a basic system according to the form of the aim system, then the difference between the output of the aim system and the output of the copy system is used as the variable to correct, and it is feedback through plus matrix to the input of the integrator in the copy system, thus, a closed-loop feedback system yield. But when the aim system has some uncertain ingredients (for example, the parameters are uncertain), the state-observer cannot copy the structure of the aim system completely.

In this paper, based on the state-observer designed, a controller $u$ is added, when an appropriate controller $u$ is chosen, not only the observation for the states, but also the synchronization of the aim system and its observation system can be realized.

The observation system of the form is as follows
\[
\dot{X} = AX + BG(X) + C + L_1(Y - \hat{Y}) + Du, \\
\dot{X}(0) = \dot{X}_0, \\
\dot{Y} = K\dot{X} + G(X),
\]
(3)

where $\dot{X}$ is the state of the observation system, $\dot{Y}$ is the output, and $L_1 \in \mathbb{R}^{n \times n}$ is unknown coefficient matrix, $D_1 \in \mathbb{R}^{n \times n}, G : \mathbb{R}^m \rightarrow \mathbb{R}^n (m \leq n), u(t) \in \mathbb{R}^n$. Obviously, the situations when the aim system has some uncertain ingredients have been included in the state-observer.

The state error system
\[
\dot{e} = \dot{X} - \hat{X} = (A - L_1K)e + (B - L_1)G(X) + (L_1 - B)F(X) + Du,
\]
(5)

where $e(0) = X_0 - \hat{X}_0$.

Let $A - L_1K = L$, then
\[
\dot{e} = Le + (B - L_1)(G(X) - F(X)) + Du,
\]
\[
e(0) = \dot{X}_0 - \hat{X}_0.
\]
(6)

It is obvious that $L$ is the part of linear in the (6), then the control of synchronization is turned into the stabilization of the error system (6) at the balance point, that is to say, searching the appropriate $L_1, B$ and $u(t)$ for all the initialization conditions of $X_0$ and $\hat{X}_0$ to satisfy
\[
\lim_{t \rightarrow \infty} = \lim_{t \rightarrow \infty} |\dot{X}(t) - X(t)| = 0,
\]
then (1) and (3) can arrive at the chaotic synchronization.

If $D$ is full rank in column, formula (6) will be written as:
\[
\dot{e} = Le + D[h(X) - l(X) + u(t)],
\]
(8)

where
\[
h = (D^TD)^{-1}D^T[B - L_1G], \\
l = (D^TD)^{-1}D^T[B - L_1F].
\]

Although chaotic synchronization can be realized with the idea of state-observer, they are different in essence. On one hand, the state-observer is designed to estimate the states of the aim system when the states can't be measured completely or partly, and it is realized by use of the input states and output states. On the other hand, the design of the chaotic synchronization system can be realized when the information of the aim system is known or can be got.

2 CHAOTIC SYNCHRONIZATION BASED ON NONLINEAR STATE-OBSERVER

Suppose 1
\[
|h(\dot{X}(t))| \leq \gamma(\dot{X}(t)), \forall \dot{X}, t, \\
|l(\dot{X}(t))| \leq \beta(\dot{X}(t)), \forall \dot{X}, t,
\]
Where $\gamma(t), \beta(t)$ is continuous.

Generally, for all the time $t$, regardless of in the balance point, period state or in the condition of chaos for the system,
\[
\|\dot{X}(t)\| \leq M_1, \forall t \in [0, \infty), M_1 \in \mathbb{R}^+, \\
\|X(t)\| \leq M_2, \forall t \in [0, \infty), M_2 \in \mathbb{R}^+,
\]
when Suppose 1 is satisfied, there exist $W, T \in \mathbb{R}^+$, such that:
\[
\|h(\dot{X}(t))\| \leq W, \|l(X(t))\| \leq T.
\]
(9)