Dynamic characteristics of a WPC—comparison of transfer matrix method and FE method

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Abstract: To find the difference in dynamic characteristics between conventional monohull ship and wave penetrating catamaran (WPC), a WPC was taken as an object; its dynamic characteristics were computed by transfer matrix method and finite element method respectively. According to the comparison of the nature frequency results and mode shape results, the fact that FEM method is more suitable to dynamic characteristics analysis of a WPC was pointed out, special features on dynamic characteristics of WPC were given, and some beneficial suggestions are proposed to optimize the strength of a WPC in design period.

Key words: Wave penetrating catamaran; Dynamic characteristics; Transfer matrix method; FEM.

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1 INTRODUCTION

Compared with conventional monohull ship, wave-penetrating catamaran has some outstanding features, such as roomy deck area, favorable stability, high speed and good seakeeping. All these advantages result from its special ship lines. Due to its favoring ship lines, some special structural problems occur, which are different from that of a conventional monohull ship. For example, the primary load that the wave penetrating catamaran is subjected to, is transverse wave induced load as mentioned in ref [1], so its primary distortion is transverse bend, and contrary to vertical bend in a conventional monohull ship. Because of the existence of two drain demi hull, length-breadth ratio of a wave penetrating catamaran is always smaller than that of a conventional monohull ship. Along most part of the length of a conventional monohull ship, its transverse section shape remains constant and is the same as that of the middle part of the ship. But the shape of a wave-penetrating catamaran changes quickly along the longitudinal direction. All these differences between wave penetrating catamaran and monohull ship bring about the different dynamic characteristics of these two types of ships.

To get the dynamic characteristics of this type of ship, only few works have been done by scholars, though many methods have been developed to solve dynamic problems of conventional monohull ship. Transfer matrix method and 3-dimension finite element method are the two typical methods. In transfer matrix method, a ship is divided into a number of parts along its longitudinal direction. And in each part of the ship, the section modulus is regarded to be equal. This is precise enough for conventional monohull ship due to the existence of the parallel body. But for wave penetrating catamaran, due to the sharp variety of the section shape along the longitudinal direction and the small length-breadth ratio, the hypothesis may not be precise and can induce prodigious error in calculation. And 3-D FEM method can express the features of an actual structure faithfully and has the advantage of high precision. In this paper, a wave-penetrating catamaran is taken as an object; its dynamic characteristics are computed by both transfer matrix method and FEM method. According to the results of natural frequencies and mode shapes, a comparison of special features of dynamic characteristics of wave penetrating catamaran and monohull ship are pointed out, applicability of these two methods to dynamic characteristics analysis on wave penetrating catamaran is evaluated, and some beneficial suggestions are proposed.
2 TRANSFER MATRIX METHOD

To get the dynamic characteristic, the ship is divided into a number of pieces. It is assumed that in each of the piece, the section variables, such as mass, inertial moment and torsion rigidity, are constant, and these variables is determined by considering every component in this pieces. By this means a ship can be treated as a beam, which is separately uniform. Using the free boundary conditions at both ends of ship hull, the nature frequencies and mode shapes can be computed by transfer matrix method.

To flexure vibration, the flexure displacement denoted as \(w\), rotate angle as \(\theta\), bending moment as \(M\) and shearing force as \(N\) are taken as the state vector in a beam piece. State vector can be simplified to matrix form: 
\[
Z(x) = \begin{bmatrix} w(x) \\ \theta(x) \\ M(x) \\ N(x) \end{bmatrix} = B(x)A,
\]

in which, A and B are coefficient matrix and matrix corresponding, its integrated expression is indicated in ref. [2].

In a beam piece, the state vector of an arbitrary point can be obtained by multiplying the state vector \(Z(x)\) of the beam end node with the field matrix \(F\). And the state vector \(Z(x)\) can be transferred between beam pieces by point matrix \(P\). It should be pointed out that, in point matrix \(P\), all the factors that affect structure's stiffness must be taken into account, such as lump mass \(M_i\), torsion inertial moment \(J_i\), translation spring constant \(K\), and torsion spring constant \(K_w\). The detailed expression of field matrix and point matrix are introduced in ref. [2].

From the characteristic of a chain-like structure, conclusion can be drawn as follows:
\[
Z_a = P_a F_a P_{a-1} F_{a-1} \cdots P_1 F_1 P_0 Z_0.
\]

Denote 
\[
\prod = P_a F_a P_{a-1} F_{a-1} \cdots P_1 F_1 P_0
\]

as transfer matrix of the whole ship hull.

The free-free boundary conditions of ship hull are taken into account, i.e., moment \(M_i\) and \(M_x\), shear force \(N_s\) and \(N_y\) are zero at the left and right end of ship hull. These conditions are substituted into expression (1) and zero state vectors are eliminated, the eigenvalue for the natural frequencies and vibration mode can be solved.

In some cases, torsion effect is required to be taken into consideration in dynamic analysis. Then the twist angle, torsion moment and warp item should be added into state vectors. The field matrix and point matrix will be modified accordingly. This has been discussed in ref. [3-4].

It has been pointed out in many previous researches that transfer matrix can give precise frequency result and mode result for conventional ship, especially for the ship with a long parallel body. But for wave penetrating catamaran whose structure features are much different from conventional monohull ship, it should be considered carefully whether this method is suitable or not.

3 FEM METHOD ON DYNAMIC CHARACTERISTIC

The advantage of a 3-D FEM model of a whole ship is that it can reflect the feature of the ship structure, the joint of components, and the distribution of the load faithfully. So it becomes a relatively advanced method for analysis of a ship structure. To verify the dynamic characteristic results obtained by transfer matrix method, a 3-D FEM model is adopted to analyze dynamic characteristic of the catamaran further, so as to ensure the integrity and veracity.

Under the stimulation of load, the FEM equilibrium equation of dynamic response of the ship structure can be expressed as:
\[
\left( \begin{bmatrix} K \end{bmatrix} + i\omega \begin{bmatrix} C \end{bmatrix} - \omega^2 \begin{bmatrix} M \end{bmatrix} \right) \{x\} = \{f_i\}.
\]

In the above function, \([K]\) is stiffness matrix, \([C]\) is damp matrix, and \([M]\) is mass matrix respectively; \(\{x\}\) is nodal displacement vector of ship FEM model; \(\{f_i\}\) is stimulant load acting on the structure. As only the nature mode of vibration of the catamaran is considered in this paper, the \(\{f_i\}\) and \([C]\) is regarded as zero during this analysis. The simplified equation of the equilibrium equation becomes as:
\[
\left( \begin{bmatrix} K \end{bmatrix} - \omega^2 \begin{bmatrix} M \end{bmatrix} \right) \phi = 0.
\]

In fact, \(\phi\) must not be zero, so we have:
\[
\det(\begin{bmatrix} K \end{bmatrix} - \lambda \begin{bmatrix} M \end{bmatrix}) = 0,
\]

(4)