A Regional Spectral Nested Multilevel
Primitive Equation Model

Liao Dongxian (廖洞贤)
State Meteorological Administration, Beijing 100081
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ABSTRACT

By means of vertical normal modes a regional nested multilevel primitive equation model can be reduced to several sets of shallow water equations characterized by various equivalent depths. Therefore, time integration of the model in spectral form can be performed in the manner similar to those used in the spectral nested shallow water equation model case.

I. INTRODUCTION

In a previous paper, the author proposed a scheme for integrating a regional spectral nested shallow water equation model, in which a method of spectral expansion in a limited area without any additional conditions, and a set of spectral equations were given.

This paper is a continuation of the previous one, with a view of extending its scheme to a multilevel primitive equation model case. For this purpose the multilevel primitive equation model has been reduced to several sets of shallow water type equations by means of vertical normal modes, in order to use the techniques in the previous paper to carry out the corresponding time integration. In the previous paper, however, the mean depth of the fluid surface, equivalent to the height of the homogeneous atmosphere, is a constant; the time step selected in the explicit case is quite small under the constraint of the linear computational stability criterion. In this paper, on the contrary, the possibility of selecting a time step larger than usual one greatly increases, owing to the equivalent characteristic depths of the shallow water type equations, except few cases, being far less than the height of the homogeneous atmosphere. For this reason, the scheme given here might save much computation time and keep reasonable accuracy for a multilevel primitive equation model.

II. VERTICAL COORDINATE, VERTICAL DISCRETIZATION AND VERTICAL BOUNDARY CONDITION

1. Vertical Coordinate

In the vertical the $\sigma$-coordinate defined as

$$\sigma = \frac{p}{p_s},$$

is adopted, where $p$ is the pressure; $p_s$ the surface pressure.

2. Vertical Discretization of the Model Atmosphere

Usually the vertical discretization of the model atmosphere can be determined from the object of prediction and objective conditions. Now we discuss a general case. It is assumed that the atmosphere is divided into $K$ layers (see Fig. 1). At each integer level the wind
components $u$ and $v$, the temperature $T$ and the geopotential height $\varphi$ are predicted, while the vertical velocity $\dot{\sigma}$ is computed at each half level.

However, the thickness of each layer $\Delta \sigma_k (= \sigma_{k+1/2} - \sigma_{k-1/2})$ may vary with $k$

but $\sum_{k=1}^{K} \Delta \sigma_k = 1$.

3. Vertical Boundary Condition

At $\sigma = 0$ and 1, the homogeneous boundary condition

$\dot{\sigma} = 0 \quad (2)$

is adopted.

3. Dynamic and Thermodynamic Equations

At $\sigma_k$-level the dynamic and thermodynamic equations in the adiabatic and inviscid case may be written as

$$\frac{\partial U_k}{\partial t} = - \frac{\partial G_k}{\partial x} + fV_k + (N_u)_k \quad (3)$$

$$\frac{\partial V_k}{\partial t} = - \frac{\partial G_k}{\partial y} - fU_k + (N_v)_k \quad (4)$$

$$\frac{\partial T}{\partial t} = - \frac{RT_k}{c_p} \left( \frac{\partial}{\partial \sigma} \frac{\dot{\sigma}}{\sigma} + D \right)_k - \dot{\sigma} \frac{\partial T}{\partial \sigma} + (N_T)_k \quad (5)$$

$$\frac{\partial P}{\partial t} = Ad_k(p) - D_k - (\delta \dot{\sigma})_k \quad (6)$$

$$\left( \frac{\partial \varphi}{\partial \ln \sigma} \right)_k = - RT_k \quad (7)$$

Utilizing the vertical boundary condition (2) and summing up the continuity equation (6) from $k=1$ to $K$ in the vertical, we have

$$\frac{\partial P}{\partial t} = \sum_{k=1}^{K} Ad_k(p) \Delta \sigma_k - \sum_{k=1}^{K} D_k \Delta \sigma_k \quad (8)$$