PARTIAL DRIFT COMPENSATION
IN ELECTRONIC D-C ANALOG COMPUTERS FOR
DIFFERENTIAL EQUATIONS

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Summary
The most serious errors in electronic d-c analog computers for differential
equations arise from drifts due to unbalances in the amplifiers. We will
describe a certain method of reducing these errors: partial drift compensa-
tion. Here the analog computer as a whole is compensated for with regard
to certain drifts, i.e. certain drifts in the way they appear in the differen-
tial equation. The sources of one drift may be located in several elements
of the computer. The number of points where the compensation has to be
applied is usually much smaller than the number of sources. Thus the par-
tial compensation is an economic solution of the drift problem if it can be
realized. This question is treated in the paper and it is shown that this kind
of compensation is well suited for analog computers with time-shared
elements. An example is finally given where the number of compensation
points is nine, although the number of drift sources is forty-five.

§ 1. Introduction. When solving a differential equation on an
analog computer it is generally not possible to avoid small differences
between the given differential equation and the equation which in
reality governs the analog. Thus a question of greatest importance is:
how do these differences affect the solution of the differential
equation? We will here investigate this question with special regard to
drift errors in electronic d-c analog computers. However, the investi-
gation is rather general and will also include other errors which do not
raise the order of the differential equation.

In general, drifts in elements for operations independent of time,
as for instance summation and multiplication, are in themselves not
dangerous. With a rather simple design of the amplifiers, the indi-
vidual errors can be kept down to the order of one pro mille of the
working range. If, however, these elements are connected with inte-
grators, i.e. elements with operations depending on time, in order to solve a differential equation, each integrator will start to integrate drift errors from preceding elements and from the amplifier in the integrator itself. These integrated errors easily amount to large values and have to be compensated for.

In the following we will investigate drifts giving errors steadily increasing with time and drifts giving limited errors. Maximum values of the drift errors will be given, and these values will decide the number of drifts which have to be compensated for.

Very often compensation of just a small number of drifts turns out to be sufficient to obtain a certain accuracy. We call this compensation partial drift compensation. The partial compensation must not be confused with ordinary compensation of a certain number of the elements in an analog computer. With partial compensation we mean instead a compensation of the analog computer as a whole with regard to certain drifts in the way they appear in the differential equation. The sources of a certain drift may be located in several elements of the computer. By partial compensation, the number of points where the compensation has to be applied is usually much smaller than the number of drift sources. Thus the partial compensation is an economic solution of the drift problem if it can be realized.

The investigation of the possibilities of partial drift compensation is the main purpose of this paper. It will be shown that this kind of compensation is well suited for analog computers with time-shared elements, i.e. elements with a common computing part, performing different operations in different time-intervals.

We will finally give an example of partial drift compensation as applied to an analog machine for computing the direction cosines between two rotating coordinate systems. This machine consists mainly of time-shared elements. Here the number of compensation points will be nine, although the number of drift sources is forty-five. The technical part of the compensation will be treated with the aid of this example.

§ 2. Incorporation of drifts in the differential equation. We start the investigation with a rather general initial value problem

\[ \frac{d}{dt} y_i = f_i(t, y_1, y_2, \ldots, y_n), \]

\[ y_i(t = 0) = y_{i0} \quad (i = 1, 2, \ldots, n). \]