LAMINAR FLOW IN CHANNELS WITH POROUS WALLS IN THE PRESENCE OF A TRANSVERSE MAGNETIC FIELD

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Summary

The steady two-dimensional laminar flow of a viscous incompressible and electrically conducting fluid through a channel of rectangular cross-section in the presence of a transverse magnetic field is discussed when the fluid is being withdrawn through both the walls of the channel at the same rate. It is assumed that the suction Reynolds number and the magnetic Reynolds number are both small. Approximate expressions for the velocity components and pressure are obtained. It is found that the pressure drop in the major flow direction decreases with the increase in suction and increases with the increase in the strength of the magnetic field. Also, the skin friction decreases with the increase in the suction or the magnetic field.

§ 1. Introduction. The steady two-dimensional flow of a viscous incompressible fluid through a channel of rectangular cross-section has been discussed by Berman 1) and Yuan 2) when the fluid is being withdrawn through the channel walls at the same rate. Whereas Berman has studied this problem for small values of suction Reynolds number, Yuan has discussed it for large values. In this paper we are discussing the steady two-dimensional laminar flow of a viscous incompressible and electrically conducting fluid through a channel of rectangular cross-section in the presence of a transverse magnetic field. The problem is treated for small values of suction Reynolds number, neglecting the induced magnetic field produced by the motion of the electrically conducting fluid.

§ 2. Basic equations. Let us consider a channel of rectangular cross-section one side of which, representing the distance between
the porous walls, is much smaller than the other. Let the $y$ axis be perpendicular to the channel walls and the $x$ axis be in a plane parallel to the channel walls with the origin at the centre of the channel. Let $L$ be the length of the channel and $2h$ the distance between the walls. The components of velocity in the $x$ and $y$ directions are represented by $u$ and $v$. A constant magnetic field of strength $H_0$ is applied perpendicular to the walls and fixed relative to them.

The equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (1)$$

The equations of momentum are

$$\rho \left( \frac{u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \sigma_e B_0^2 u, \quad (2)$$

$$\rho \left( \frac{v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), \quad (3)$$

where $\rho$ is the density of the fluid, $\mu$ the coefficient of viscosity, $p$ the pressure, $\sigma_e$ the electrical conductivity, $B_0 = \mu_e H_0$ the electromagnetic induction, $\mu_e$ being the magnetic permeability. In (2) and (3), the secondary effects of magnetic induction are ignored $^3$. Also, we shall assume that the electric field is zero; this assumption is justified since no external electric field is applied and the effect of polarisation of the ionized fluid is negligible $^4$.

In view of these assumptions Maxwell’s equations become superfluous.

Introducing $\lambda = y/h$, the equations (1)–(3) become

$$\frac{\partial u}{\partial x} + \frac{1}{h} \frac{\partial v}{\partial \lambda} = 0, \quad (4)$$

$$u \frac{\partial u}{\partial x} + \frac{v}{h} \frac{\partial u}{\partial \lambda} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 u}{\partial \lambda^2} \right) - \frac{\sigma_e B_0^2 u}{\rho}, \quad (5)$$

$$u \frac{\partial v}{\partial x} + \frac{v}{h} \frac{\partial v}{\partial \lambda} = -\frac{1}{h \rho} \frac{\partial p}{\partial \lambda} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 v}{\partial \lambda^2} \right), \quad (6)$$

where $\nu$ is the kinematic viscosity.