INDUCTION BY AN OSCILLATING MAGNETIC DIPOLE OVER A TWO LAYER GROUND

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Summary
An expression for the mutual electromagnetic coupling between two small loops over a two-layer ground is derived. The result is expressed in a form which is suitable for calculation by a digital computer. When the heights or separation of the loops are large compared to the skin depth in the ground, simple asymptotic formulas for the fields can be developed. The results are employed to obtain a convenient formula for the self-impedance of a loop over a two-layer ground.

§ 1. Introduction. It is the purpose of this note to extend some of the available formulae for the fields of an oscillating magnetic dipole over a two-layer ground 1). Both the vertical and horizontal dipoles are considered. The distance between the source dipole and the observer is much less than a free space wavelength.

It is shown that the magnetic field components can be expressed conveniently in terms of three basic integrals $T_0$, $T_1$, and $T_2$ in analogy to the case for the homogeneous ground 2). These are in suitable form for numerical integration. Asymptotic expansions for these integrals are developed. The results are used to calculate the self impedance of an arbitrarily oriented loop over a two-layer ground.

§ 2. Formulation. Choosing a rectangular coordinate system $(x, y, z)$, the source loop can be represented as a magnetic dipole of strength $C$ at $(0, 0, h)$ with respect to the ground defined for all negative values of $z$. The ground is homogeneous between $z = 0$ and $z = -d$ with conductivity $\sigma_1$ and dielectric constant $\varepsilon_1$. The region below $z = -d$ is also homogeneous with constants $\sigma_2$ and $\varepsilon_2$. The permeability of the whole space is taken $\mu_0 (= 4\pi \times 10^{-7})$.

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Subject to the limitation that all significant distances in the upper half-space are to be much less than a free space wavelength, the fields for \( z > 0 \) are a solution of Laplace's equation and hence derivable from a scalar potential. On the other hand, the fields in the lower regions are solutions of the appropriate wave equation. The validity of this quasi-static approach was previously investigated 1).

For a vertical or \( z \) directed dipole at \((0, 0, h)\) the magnetic fields for \( z > 0 \) can be written

\[ H = - \nabla \Phi^v, \tag{1} \]

where

\[ \Phi^v = - \frac{\partial}{\partial z} \left[ \frac{1}{r} + \int_0^\infty R(\lambda) e^{-\lambda(z+h)} J_0(\lambda \rho) d\lambda \right] \mathbf{C}, \tag{2} \]

with \( r = \left[ \rho^2 + (z-h)^2 \right]^{1/2}, \rho = (x^2 + y^2)^{1/2}, \)

\[ R(\lambda) = \frac{(u_1 + \lambda) (u_1 - u_2) e^{-2u_1 \delta} - (u_1 - \lambda) (u_1 + u_2)}{(u_1 + \lambda) (u_1 + u_2) - (u_1 - \lambda) (u_1 - u_2) e^{-2u_1 \delta}} . \tag{3} \]

and

\[ \gamma_1^2 = i\sigma_1 \mu_0 \omega - \epsilon_1 \mu_0 \omega^2, \quad \gamma_2^2 = i\sigma_2 \mu_0 \omega - \epsilon_2 \mu_0 \omega^2. \]

For the case of a horizontal or a  \( y \) directed dipole, the fields are obtained from

\[ H = - \nabla \Phi^h, \tag{4} \]

where

\[ \Phi^h = - \frac{\partial}{\partial y} \left[ \frac{1}{r} - \int_0^\infty R(\lambda) e^{-\lambda(z+h)} J_0(\lambda \rho) d\lambda \right] \mathbf{C}, \tag{5} \]

with \( r \) and \( R(\lambda) \) as defined above.

The three cartesian components of the magnetic field in the upper half-space for both vertical and horizontal components can be expressed in terms of three basic integrals \( T_0, T_1 \) and \( T_2 \) which are functions of the dimensionless quantities \( A, B \) and \( k \) which are defined by

\[ A = (z+h)/\delta, \quad B = \rho/\delta \]

where

\[ \delta = \left[ \frac{2}{(\sigma_1 + i\omega \epsilon_1) \mu_0 \omega} \right]^{1/2}, \quad k = \frac{\sigma_2 + i\omega \epsilon_2}{\sigma_1 + i\omega \epsilon_1}. \]