ABSTRACT. We consider the Boltzmann equation for inelastic particles on the line and prove some preliminary results on existence and uniqueness of the solutions. We also discuss some connections with another kinetic equation investigated by the same authors.

1 The model

In recent times particle systems interacting via inelastic collisions have provoked an increasing interest due to the fact that they constitute a simple mathematical model for granular media, see e.g. Ref.s [5,7,10,11,12,13,14,15,17] for preliminary physical considerations on the behavior of such systems. Unfortunately very few rigorous results are known. In this paper we attempt a mathematical study in the simple one dimensional case.

Let us consider a system of $N$ particles in $\mathbb{R}$. Let $x_i, v_i \in \mathbb{R}$ be the position and the velocity of the i-th particle and

$$Z^N = (X^N, V^N) = (x_1, v_1, ..., x_N, v_N)$$

a state of the system. The dynamics is the following. The particles move freely up to the first instant in which two of them are in the same point. They collide according to the following rule:

$$v' = v - \epsilon(v - v_1)$$

$$v'_1 = v_1 + \epsilon(v - v_1),$$

$v', v'_1$ and $v, v_1$ are the outgoing and ingoing velocity respectively, and $\epsilon \in [0, 1/2]$ is a parameter measuring the degree of inelasticity of the collision.
Note that the collision preserves the total momentum and dissipates the kinetic energy. Moreover for $\varepsilon = 0$ we have the free particle system, while for $\varepsilon = 1/2$ we have the so called sticky particle model in which the particle pair remains attached after the collision.

A relevant qualitative feature of the systems is the possibility of delivering collapses in a finite time (for a suitable values of $\varepsilon$). Indeed it can be shown that, if $\lambda = \varepsilon N$ is sufficiently large, all the particles can reach the same position and have the same momentum after a finite time and an infinite number of collisions. See Ref. [9] for the case $N = 3$ and Ref. [2] for general $N$.

The dynamics of the system is certainly complex so that, in analogy with the standard theory of rarefied gases, it is natural to derive a reduced description given in terms of a Boltzmann equation. Obviously such a description will have a limited range of validity but, for the moment, we shall disregard this fundamental aspect.

Standard arguments of kinetic theory will lead us to consider the following equation for the unknown $f = f(x, v, t)$ that is the probability density of a single particle:

$$
\partial_t f(x, v, t) + \partial_x f(x, v, t) =
\int dV f(x, v_1, t) \left( \frac{f(x, v_1, t)f(x, v_1^*, t)}{(1 - 2\varepsilon)^2} - f(x, v, t)f(x, v_1, t) \right),
$$

where $v^* = v + \frac{\varepsilon}{1 - 2\varepsilon} (v - v_1)$, $v_1^* = v_1 - \frac{\varepsilon}{1 - 2\varepsilon} (v - v_1)$, are the pre-collisional velocity and $l > 0$ is the mean free time inverse.

How to justify the introduction of this equation on the basis of logically well founded arguments? One can say that Eq. (1) is a simplified model of the more difficult two and three dimensional Boltzmann equation for rarefied gas of inelastic balls in the so called Boltzmann-Grad limit (see e.g. Ref. [8]). One the other hand, as we shall see in the following, Eq. (1) can be directly derived in terms of a stochastic systems of inelastic particles.

We also note that in Ref. [4] we have obtained another kinetic equation describing the particle system in a mean field limit. This equation read as:

$$
(\partial_t + v \partial_x) f(x, v, t) = -\lambda \partial_v (Ff)(x, v, t),
$$

where:

$$
F(x, v, t) = \int d\tilde{v}(\tilde{v} - v)|\tilde{v} - v| f(x, \tilde{v}, t).
$$

It is not difficult to show, formally, that Eq. (1) tends to Eqs (2) - (3) in the limit $\varepsilon \to 0, l \to \infty, l\varepsilon \to \lambda$

We notice that the homogeneous Eq. (2) with a Fokker-Plank term simulating a reservoir at a constant temperature, has also been studied in a