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Axiomatic Definition and Comparison of Three Aggregation Rules in Cardinal Social Choice Theory

1. Introduction and Survey

In order to get beyond K. Arrow's general possibility theorem, one way of altering his model is to assume the individual utility valuations to be both cardinal and interpersonally comparable. These assumptions are strong, but they yield far reaching results. The individual utility functions can be aggregated by means of different methods all satisfying the requirements of democracy, rationality and generality as stated by Arrow.

Two methods are particularly interesting: the utilitarian rule of maximizing the sum of the individuals' utilities and the so called maximin rule based upon the theory of justice developed by Rawls [1971]. Both rules are democratic, rational, and generally applicable methods of social choice processes. They represent, however, quite different basic concepts and can, moreover, yield different or even opposite results, thus challenging comparative analysis and judgement (see, e.g. Arrow [1973], Cooter [1975], Harsanyi [1975], Sen [1970, 1974a,b,c]).

This paper contributes to the ongoing discussion. Its objective is to develop a broader basis for comparative analyses. Its means is to characterize social choice rules uniquely by axioms representing the assumptions of the underlying theories. Among the comparisons of utilitarianism and the maximin concept there are only a few following an axiomatic approach. This paper enlarges contributions by Sen [1974b, 1976] and D'Aspremont/Gevers [1975]. Besides, it obtains help from outside of social choice theory: because of the formal equivalence between the models of cardinal social choice and individual decision making under uncertainty, some results of Milnor's [1954] axiomatic comparison of decision criteria can be utilized.

Following this introduction the second and third section contain the formal characterization of social choice rules. In addition to the utilitarian and the maximin rule the so called lexicographic maximin rule is also considered. Part 2 contains the formal representation of the cardinal social choice model, whereas the axiomatic definitions are stated and proved in section 3.

In the concluding section 4 the axioms are used in clarifying some issues of the discussion between utilitarians and 'Rawlsians'. It is shown that, for lack of generally accepted and equally interpreted criterions, the comparability of utilitarianism and Rawls's concept is limited. It is, however,
also indicated which direction further research should follow.

2. The Cardinal Model of Social Choice

2.1 Constituents of the Model

As a formal basis for the following analyses we introduce the cardinal model of social choice by its constituents:

(a) There are n participating persons or individuals, identified by 1,2,...,n. Every i is an element of the set N of individuals.

The n individuals are concerned with m alternatives representing social or economic states, candidates to be elected, etc. They are combined in the set X of states. We assume that there are at least two individuals and three states, and that both N and X are finite sets.

(b) Each individual's valuation of the states x∈X is represented by a utility function u_i: X→R, u_i(x) being the amount of utility assigned to state x by individual i. These n individual utility functions (IUF's) are unique up to positive linear transformations. A combination of n IUF's - one for each individual - is denoted by u(x) = (u_1(x),...,u_n(x)), x∈X, and called a utility profile or simply profile on X. If only a subset S⊆X is considered we speak of a profile restriction on S. Every utility profile on X is an element of the set U of all logically possible combinations of all logically possible IUF's.

(c) The objective of the social choice process, i.e. the common valuation of the states x∈X is assumed to be a weak preference ordering on X. This social preference ordering (SPO) is reflexive, transitive and complete. It is denoted by (X,R), where R is the weak preference relation ("as least as good as"). The corresponding strict preference relation is P, the relation of indifference is I. All possible weak orderings (X,~) of X make up the set (X,~).

(d) The social choice process itself is a functional relation f assigning a unique SPO to every utility profile: f: U→(X,~), f(u) = (X,R). It is called a generalized social welfare function (GSWF).

It should be noted that this definition already contains two assumptions frequently used in social choice analyses. On one hand the domain U of f is unrestricted; on the other hand the co-domain (X,~) is restricted to the set of weak orderings.

2.2 Concepts of Interpersonal Comparability

Assuming cardinality provides more information. Every IUF contains two types of information about the person's utility valuation. On one hand, a weak ordering of the elements of X represents his comparative judgement of the states according