MATRIX RELATIONSHIPS BETWEEN FAR-ZONE PATTERNS AND CURRENT DISTRIBUTIONS OVER THE PROLATE SPHEROID *

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Summary

A matrix relationship is found between the coefficients of the expansion of the far-zone radiation patterns and the coefficients of the expansion of the corresponding current distribution over the prolate spheroid. In particular cases, the matrix becomes diagonal, giving a one-to-one correspondence between the coefficients. The same formulation may be applied in the limiting case for the corresponding problems in a sphere.

§ 1. Introduction. The problem of finding a current distribution on a given surface of revolution which will produce a prescribed far zone radiation pattern is one of considerable practical importance. The mathematical formulation of such problems in terms of integral equations is well known, as is the general theory of their solution by means of series expansions 1). The difficulty incurred in general is in carrying out the necessary quadratures, i.e., evaluating the integrals involved and/or calculating the special functions or coefficients required. It develops, however, that in certain coordinate systems one can use expansions for the known and unknown functions which yield known forms for the integrals which appear, and thus the problem is reducible in general to a matrix relation between the coefficients in the expansions of the known and unknown functions. The cases dealt with in the following are typical examples of this circumstance.

In 1937 Wolff 2) published his method of obtaining any arbitrary

*) This paper was written while the author was a Consultant to the Radiation Laboratory, University of Michigan, Ann Arbor, Michigan, during the summer, 1958.
far-zone circularly symmetric pattern from radiators with uniform distribution along an array axis. His theory was based upon comparison of the far zone field of a pair of radiators to a term of the Fourier series expansion of the prescribed pattern. Further very important contributions to the subject were given by Schelkunoff and Dolph and their results are well known. Matrix relationships for arrays with arbitrarily distributed elements have been found for linear and multi-dimensional arrays. The problem of determination of the current distribution over a cone surface which will produce a prescribed radiation pattern has been solved as described above for the case of currents polarized in the direction of the generating lines of the cone.

Matrix relationships will be developed in the present paper for currents distributed over a prolate spheroid and their relationship with the corresponding far zone radiation patterns. It must be noted here that the sphere case may be developed directly or as a particular case of the present discussion of the prolate spheroid. It should be mentioned that in all cases we use the same notation for the coefficients for simplicity. However, those coefficients are different for each section except for particular cases. The relationships found in this paper have not been written formally in matrix notation. This formal notation, as well as additional discussion, may be found elsewhere.

The method of excitation of the required current distribution over the surfaces is not discussed here. It is assumed that once the current distribution has been found, it can be excited over the surface. It is known that the problem of finding the current distribution over a surface which will produce a prescribed radiation power is not unique. There is an infinite set of current distributions which will produce, within a certain approximation, the same far zone pattern.

The far zone radiation patterns will be defined as follows:

\[ F(\theta; \phi) = \int \int_S (K \cdot i_\theta) e^{ika} i_\phi \, dS, \]  
\[ G(\theta; \phi) = \int \int_S (K \cdot i_\phi) e^{ika} i_\theta \, dS, \]

where \( i_R; i_\theta; i_\phi \) are the unit polar coordinate vectors at the far zone observation point \( P(R; \theta; \phi) \), \( K \) is the surface current (electric or magnetic), \( a \) is position vector from the origin to the source point...