A Comparative Study of Default Reasoning and Epistemic Processes

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Abstract

A comparative study between the theories of default reasoning and open logic is given. Some concepts of open logic, such as new premises, rejections by facts, reconstructions, epistemic processes, and its limit are introduced to describe the evolution of hypotheses. An improved version of the limit theorem is given and proved. A model-theoretic interpretation of the closed normal defaults is given using the above concepts and the corresponding completeness is proved. Any extension of a closed normal default theory is proved to be the limit of a $\delta$-partial increasing epistemic process of that theory, and vice versa. It is proved that there exist two distinct extensions of a closed normal default theory iff there is an $\delta$-non-monotonic epistemic process of that theory. The completeness of Reiter's proof is also given and proved, in terms of the epistemic processes. Finally, the work is compared with Gärdenfors's theory of knowledge in flux.

Keywords: Default reasoning, epistemic processes, open logic.

1. Introduction

The goals of many non-monotonic approaches concerning common sense reasoning (for example, [7], [8], [9] and [11]) are:

1. To build one formal system which is either a classical or a modal logic system, plus a mechanism for describing aspects such as incompleteness of information, the fallibility of knowledge, and the non-monotonicity of reasoning.
2. To use these mechanisms for predicting events which happen in the real world, according to the obtained incomplete and fallible information.

In fact, there exists an alternative way to look at these aspects: Incompleteness of information is revealed when some "new" (which will be defined precisely in Section 2) facts are obtained, and incomplete information becomes less incomplete in an epistemic process of a hypothesis. The fallibility of a hypothesis cannot be realized when the hypothesis is first devised. They are detected later, when some experiments and observations contradict the hypothesis, or a counter-example is found. The non-monotonicity arises when some fallibility are rectified during the evolution of a hypothesis.

A theory of epistemic processes in [5] is set up, following this point of view. It is a theory of the sequences of hypotheses (- or belief sets) that can model the evolution of hypotheses. It introduces concepts such as the new premises and the rejections by facts to describe the interaction between logical information (logical inferences) and empirical information (observations), the revisions of a hypothesis to express the improvement of knowledge, and the epistemic processes to represent the evolution of a hypothesis.

The purpose of this paper is to give a comparative study between the theories of default reasoning and epistemic processes. Since the latter has a sound model theory, the study leads to a model-theoretic interpretation of the former. Before going to the technical details, let us look at the example below:
Example 1.1. Mechanics.

Consider Galileo's principle of the addition of velocity: "If, relative to a coordinate system $K$, $K'$ is a moving coordinate system with a uniform velocity $w$ without rotation, and a body is moving in $K'$ with a velocity $v$, then the velocity of the body with respect to $K$ is $v + w$."

Assume that the formal language used in this paper is a first order language. Let $B(x)$ denote " $x$ is a body ", and let $A(x)$ denote " the velocity of $x$ with respect to $K'$ depends on both the velocity $w$ of $K'$ with respect to $K$ and the velocity $v$ of the body with respect to $K$ (and is $w + v$) ". Thus, the principle of addition of velocity is represented as:

$$V : \forall x . (B(x) \Rightarrow A(x))$$

The viewpoint of Reiter's default reasoning is:

The principle $V$ can be represented as a default: 

$$B(x); M, A(x)$$

It means that if $x$ is a body and $A(x)$ (Galileo's principle of the addition of velocity) does not conflict with the knowledge of mechanics which we know, then we accept $A(x)$.

For example, let $\Gamma$ be the knowledge which Galileo knew. Then, since $A(x)$ does not conflict with $\Gamma$, $A(x)$ is acceptable with respect to $\Gamma$.

Let us consider another case. For the purpose of this discussion, we assume light $c$ to be of particulate nature, i.e. $B(c)$ holds. According to human experiments and astronomical observations, we accept the following statement: "At all events we know with great exactness that ... the velocity of propagation of light cannot depend on the velocity of motion of the body emitting the light" ([1]), i.e. $\neg A(c)$ holds. Let $\Gamma'' = \{ \Gamma, B(c), \neg A(c) \}$. For light $c$, $A(c)$ does contradict $\Gamma''$. Thus $A(c)$ (Galileo's principle) is not accepted with respect to $\Gamma''$.

In contrast, the viewpoint of epistemic processes is:

Assume that $\Gamma$ denotes the knowledge of the mechanism known by Galileo, and that $\Gamma$ represents the first stage of an epistemic process about the mechanics (for example, the mechanics in Galileo's time).

Since $V$ does not conflict with $\Gamma$ (in fact, $V$ is even logically independent of $\Gamma$), Galileo added it into his mechanics as a new law. In this paper, we call $V$ a new premise for $\Gamma$.

Assume that $c$ is of a particle nature. Let $\Gamma' = \{ \Gamma, V, B(c) \}$ represent the second stage of the epistemic process of the mechanics (for example, the mechanics just before Einstein's special relativity theory). It is obvious that $A(c)$ can be deduced from $B(c)$ and $V$. Using a Gentzen style notation ([2]), this can be represented as:

$$\Gamma' \vdash A(c)$$

It means that the theory $\Gamma'$ predicts that the velocity of propagation of light depends on the velocity of the body emitting the light.

Then, scientists discovered that all experiments and astronomical observations reject this prediction (as we quoted above). Let us call it a rejection of $A(c)$ by facts. Clearly, $\Gamma'$ must be revised to match the experiments and observations while maintaining consistency between laws. A simple solution is to accept $\neg A(c)$ as a new law. Since $B(c), \neg A(c) \models \neg(\forall x . B(x) \Rightarrow A(x))$

$\neg A(c)$ is not consistent with $\forall x . B(x) \Rightarrow A(x)$. We have to refute Galileo's principle. Thus we have $\Gamma'' = \{ \Gamma, B(c), \neg A(c) \}$. $\Gamma''$ represents the third stage of the epistemic process of mechanics (for example, Einstein's special relativity theory ([1])).

The sequence of hypothesis: $\Gamma_1, \Gamma_2, \Gamma_1, \ldots$ describes the evolution of mechanics. We call the sequence an epistemic process of mechanics. $\Gamma_1 \subseteq \Gamma_2$ and $\Gamma_2 \subseteq \Gamma_3$ indicate that non-monotonicity is a characteristic of the epistemic processes. when rejections of facts arise.

The rest of the paper is organized as follows. In Section 2, the model-theoretic concepts of new premise, rejection by facts and epistemic processes are defined; their corresponding proof-theoretic concepts are also studied. In Section 3, the definition of the limit of a sequence of