Neural Network Methods for NURBS Curve and Surface Interpolation

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Abstract

New algorithms based on artificial neural network models are presented for cubic NURBS curve and surface interpolation. When all the knot spans are identical, the NURBS curve interpolation procedure degenerates into that of uniform rational B-spline curves. If all the weights of data points are identical, then the NURBS curve interpolation procedure degenerates into the integral B-spline curve interpolation.

Keywords: Neural network, NURBS, curve interpolation, surface interpolation.

1 Introduction

The nonuniform rational B-splines (NURBS)[1] are versatile tools used for curve and surface modeling. A growing number of NURBS-based CAD/CAM systems have been developed. In practice NURBS curve interpolation problems are often encountered, especially when we have data points that are related to other entities such as section curves from other processes. Another problem of NURBS surface interpolation is to construct a smooth surface interpolating to 3D data sets. Although the global B-spline curve interpolation problem can be solved easily[2-5], NURBS curve interpolation is a very difficult problem. Farin[3] introduced a solution to the interpolation problem in the context of rational B-splines, in which the problem of how to choose the weights for the control vertices was not addressed, and even though all of the weights of the data points are positive, it cannot guarantee positive weights for all control vertices, so that the denominator of the interpolation equation may be zero. This is one of the research problems on NURBS currently being considered[1,3,6]. A solution to this problem can be transformed into a quadratic programming problem[7]. In this paper, a new algorithm for cubic NURBS curve interpolation is presented. It is based on artificial neural network models, in which there has been a recent resurgence caused by new net topologies and algorithms, analog VLSI implementation techniques, and the belief that massive parallelism is essential for high performance speech and image recognition, graphic generation and geometric modeling. The algorithm can also be used for NURBS surface interpolation.

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In the paper we assume that weights of control vertices of NURBS curves and surfaces are always positive since the negative weights have some serious drawbacks\cite{1}.

2 Cubic NURBS Curve Interpolation

2.1 Interpolating 3D Points with a Cubic NURBS Curve

The interpolation problem in the context of cubic NURBS curve is the following:

Given: 3D data points $P_0, P_1, \ldots, P_n$ and weights $w_0, w_1, \ldots, w_n$;

Find: a $C^2$ NURBS curve with control vertices $V_0, V_1, \ldots, V_{n+2}$ and weights $W_0, W_1, \ldots, W_{n+2}$ that interpolates to the given data points and weights.

Usually, it seems reasonable to assign high weights in regions where the interpolant is expected to curve sharply. When the interpolated curve is wavelike, the given weights may be undulatory. In homogeneous coordinate system, a lot of algorithms for cubic nonuniform B-splines\cite{3-5} can be used directly. Unfortunately, when they are applied to the 4D homogeneous coordinate system directly, even though all of the weights of the data points are positive, the positive weights for the control vertices\cite{4} cannot be guaranteed, as the spline interpolation can exhibit undulations, and the denominator of the NURBS formula may be zero. In order to prevent such cases, new methods for determining the weights of control vertices are presented based on neural network in this section.

2.1.1 Knot Vector

Determining the knot vector of an interpolating NURBS curve can be described as follows:

Given: $P_i, w_i, i = 0, \ldots, n$;

Find: knot vector $\mathbf{T} = \{t_i\}$.

Using the centripetal method\cite{11}, the knot vector of the nonperiodic NURBS curve of degree three can be calculated by the following formula:

$$\mathbf{T} = [t_0, t_1, \ldots, t_p, t_{p+1}, t_{n+p-1}, t_{n+p}, \ldots, t_{n+2p}]$$  \hspace{1cm} (1)

where

$$\begin{align*}
&\begin{cases}
  p = 3, \\
  t_0 = t_1 = \cdots = t_p = 0, \\
  t_{n+p} = t_{n+p+1} = \cdots = t_{n+2p} = 1
\end{cases} \\
&t_{p+i} = t_{p+i-1} + \frac{\sqrt{|P_i - P_{i-1}|}}{\sum_{j=1}^{n} \sqrt{|P_j - P_{j-1}|}}; \hspace{1cm} i = 1, 2, \ldots, n-1
\end{align*}$$  \hspace{1cm} (2)

2.1.2 Weights

The normalized nonuniform B-spline basis functions of degree $p$ are defined by the following de Boor-Cox recursive formulae\cite{11}:

$$N_{i,0}(t) = \begin{cases} 
  1, & t \in [t_i, t_{i+1}) \\
  0, & t \notin [t_i, t_{i+1})
\end{cases}$$