On the Characterization and Fault Identification of Sequentially $t$–Diagnosable System Under PMC Model

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Abstract

Sequential diagnosis is a very useful strategy for system-level fault identification because of its lower cost of hardware. In this paper, the characterization of sequentially $t$–diagnosable system is given, and a universal algorithm to seek faulty units in the system is developed.

1. Introduction

With the advent of multiprocessors and computer networks, it is possible to increase the system reliability by means of system-level fault detection and identification. Since the technique of system diagnosis was introduced by Preparata, Metze and Chien in 1965 [1], considerable progress has been made on it [2]. Sequential diagnosis is a very useful fault identification strategy because of its lower cost of hardware. So, investigation on this kind of diagnosable system is of realistic significance.

However, the previous researches mainly focused on devising some optimal systems and algorithms for finding faulty units in special cases. The characterization of sequentially $t$–diagnosable system problem has not been developed yet, and no universal algorithm for determining faulty units has been presented. The goal of this paper is to study these two topics.

2. Preliminaries

A system $S$ consists of $n$ units, denoted by set $U = \{ u_1, u_2, \ldots, u_n \}$, the status of each unit can be distinguished by one corresponding boolean variable. If the faults in units to be considered are permanent, assume that $x_i = 0$ means the unit $u_i$ is normal, $x_i = 1$ indicates $u_i$ is faulty. The syndrome $W$ which contains outcomes of all tests in system is a 0–1 boolean constant vector, $w_{ij} = 0$ ($1$) in $W$ shows $u_i$ evaluates $u_j$ to be fault free (faulty). Under PMC model, the test outcome $w_{ij}$ is reliable if and only if $u_i$ is fault free. Let $X$ be an $n$-dimension boolean variable vector, then $X$ and $W$ satisfy the following boolean equation group (2.1) [9],

$$x_i = x_i + \sum_{j \in I(x_i)} (x_j \oplus w_{ij}) \quad i = 1, 2, \ldots, n,$$

where $I(x_i) = \{ j \mid \text{if there exists one test from } u_i \text{ to } u_j \}$. Each equation in (2.1), say $x_i = x_i + \sum_{j \in I(x_i)} (x_j \oplus w_{ij})$, shows that every unit in the set $U(\{ u_i \})$ $(U(\{ u_i \}) = \{ u_j \mid j \in I(x_i) \})$ is tested by unit $u_i$. And the equation group (2.1) composed of $n$ equations reflects the diagnosis construct of system and the logic relations between $X$ and $W$. For example, the equation group for the system given in Fig. 1 is:

$$
\begin{align*}
x_1 &= x_1 + (x_2 \oplus w_{12}) \\
x_2 &= x_2 + (x_3 \oplus w_{23}) \\
x_3 &= x_3 + (x_1 \oplus w_{31})
\end{align*}
$$

Fig. 1
Throughout this paper, the following definitions and notations are used.

For two boolean constants $x_i, y_j$, the operation $x_i \oplus y_j$ is defined as $x_i \oplus y_j = x_i \cdot \overline{y_j} + \overline{x_i} \cdot y_j$.

Let $X$ and $Y$ be two boolean constant vectors, and $X = (x_1, x_2, \ldots, x_n)$, $Y = (y_1, y_2, \ldots, y_n)$. Some operations are defined as follows:

$Z = X \oplus Y$, where $z_i = x_i \oplus y_j$;
$Z = X + Y$, where $z_i = x_i + y_j$;
$Z = X \cdot Y$, where $z_i = x_i \cdot y_j$.

$\mathcal{F}(X)$ is a subscript set, $\mathcal{F}(X) = \{ j | x_j = 1 \text{ in } X \}$. $X \cdot Y = 0$ means each element of $X \cdot Y$ is equal to zero. $\|X\|$ is the number of elements which equals 1 in $X$.

The set $\mathcal{K}$ is defined as:

$\mathcal{K} = \{ X | \| X \| \leq t \text{ for each } n \text{-dimension boolean constant vector } X \}$, that is $\mathcal{K}$ includes all 0-1 vectors in which the number of elements being 1 is equal to or less than $t$.

Given a syndrome $W$, let $X(w)$ denote a solution vector of boolean equation group (2.1) with respect to $W$. Let $\mathcal{K}(w) = \{ X(w) | \| X(w) \| \leq t \text{ for each solution } X(w) \text{ of the equation group with respect to } W \}$.

### 3. The Characterization of Sequentially $t$-Diagnosable System

**Definition 3.1.** A system is sequentially $t$-diagnosable if and only if at least one faulty unit can be identified without replacement provided the number of faulty units does not exceed $t$.

**Lemma 3.1.** A system is sequentially $t$-diagnosable if and only if for arbitrary given syndrome $W$, it has $\mathcal{K}(w) = \Phi$ or $X_1(w) \cdot X_2(w) \cdot \ldots \cdot X_p(w) \neq 0$, where $\mathcal{K}(w) = \{ X_1(w), X_2(w), \ldots, X_p(w) \}$.

**Proof.** $\Rightarrow$ For any given syndrome, if $\mathcal{K}(w) = \Phi$, then it means no faulty unit occurred, which satisfies Definition 3.1.

If $\mathcal{K}(w) = \{ X_1(w), X_2(w), \ldots, X_p(w) \} (p \geq 1)$, and $X_1(w) \cdot X_2(w) \cdot \ldots \cdot X_p(w) \neq 0$, then there exists at least a common element in these solution vectors, say $x_i$, and $x_i = 1$, this shows that $u_i$ must be faulty. Thus, one faulty unit is found, which matches Definition 3.1.

$\Leftarrow$. If there exists one syndrome $W$ such that $\mathcal{K}(w) = \{ X_1(w), X_2(w), \ldots, X_p(w) \}$, then this means there is no common element equals 1 in these solution vectors, so none of the faulty units can be determined from this syndrome, which does not satisfy Definition 3.1. Thus, the system is not sequentially $t$-diagnosable. Q.E.D.

**Lemma 3.2.** Given a syndrome for system $S$, if $X$, $Y$ are two solutions of equation group (2.1) with respect to $W$, then $X + Y$ is a solution for the same syndrome also.

**Proof.** Let $Z = X + Y$, put it into boolean equations (2.1), then we can easily verify that it satisfies (2.1) also. Q.E.D.

**Lemma 3.3.** Given arbitrary $k (k \geq 2)$ distinct vectors $X_1, X_2, \ldots, X_k$ belonging to $\mathcal{K}$, if for any two different vectors $X_p, X_q \in \{ X_1, X_2, \ldots, X_k \}$ such that no equation $x_i = x_i + \ldots + (x_j \oplus w_{ij}) + \ldots$ in (2.1) satisfies:

$$i \notin \mathcal{F}(X_p + X_q), \quad j \in \mathcal{F}(X_p + X_q).$$

then $X_1, X_2, \ldots, X_k$ must be in $\mathcal{K}(w)$ for a certain syndrome $W$.

**Proof.** By reduction, suppose $X_1, X_2, \ldots, X_k$ satisfy the above condition.

When $k = 2$, we construct a syndrome $W$ as follows: $w_{ij} = 0$ for all $i, j \notin \mathcal{F}(X_1 + X_2)$, or $i, j \notin \mathcal{F}(X_1 + X_2)$, or $i \notin \mathcal{F}(X_1 - X_2)$ and $j \notin \mathcal{F}(X_2 - X_1)$, or $i \notin \mathcal{F}(X_1 - X_2)$ and $j \notin \mathcal{F}(X_2 - X_1)$. It can be easily verified that $X_1, X_2$ belong to $\mathcal{K}(w)$ with respect to this syndrome $W$. Therefore, when $k = 2$, the proposition is correct.

Assume that when $k = r - 1$ the proposition is correct, now we show that when $k = r$ it is correct also.

Let $Y = X_1 + X_2 + \ldots + X_{r-1}$. According to the assumption of reduction, $X_1, X_2, \ldots, X_{r-1} \in \mathcal{K}(w)$ for a certain syndrome $W$. From the condition of this lemma, for each equation in (2.1), say $x_i = x_i + \ldots + (x_j \oplus w_{ij}) + \ldots$ does not satisfy $i \notin \mathcal{F}(X_p + X_q)$ and $j \in \mathcal{F}(X_p + X_q)$ for each pair of vectors.