OBLIQUE SHOCK WAVES IN DUSTY GAS SUSPENSIONS

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The equations governing the flow field which is developed when a supersonic dusty-gas suspension passes through a straight oblique shock wave were formulated. A computer code for solving the governing equations was developed and used to obtain the solution for a variety of different initial conditions. In addition, the dependence of the post-shock suspension properties on the various physical properties of the dust particles, (namely the diameter of the dust particles, their specific heat capacity, their material density and the loading ratio of the dust in the suspension) was investigated.

Key Words: Gas Particle Suspension, Shock Wave, Gaseous Phase

NOMENCLATURE

A : Envelope area of the solid particle = \pi D^2
C : Specific heat capacity of the solid particle
C D : Drag coefficient
C P : Specific heat capacity at constant pressure of the gaseous phase
C V : Specific heat capacity at constant volume of the gaseous phase
D : Diameter of the solid particle
F D : Drag force
k : Thermal conductivity
M : Flow Mach number
m p : Mass of a solid particle
N U : Nusselt number
n : A co-ordinate normal to the oblique shock wave
n p : Number density of the solid particles
P : Suspension pressure
P r : Prandtl number
Q H . T : Rate of heat per unit volume transferred from the gaseous to the solid phase
R : Specific gas constant
R e : Reynolds number
S : Cross section of the solid particle = \pi D^2/4
T : Temperature
u : Velocity of gaseous phase
v : Velocity of the solid phase
y : Specific heat capacities ratio = C P / C V
\phi : Angle of incidence
\beta : Flow deflection angle
\mu : Dynamic viscosity
\rho : Spatial density
\sigma : Material density
\eta : Loading ratio of the solid phase in the suspension
= \rho p o / \rho g o

Subscripts

s : Gaseous phase
n : Normal component
p : Solid phase
s : Tangential component
x : Horizontal co-ordinate
y : Vertical co-ordinate
o : Flow state ahead of the shock wave
l : Flow state behind the shock wave

1. INTRODUCTION

The interest in the gas-dynamic behaviour of a gas-particle suspension grew in the past three decades due to its application to many engineering problems. Some typical examples are: metallized propellents of rockets, jet-type dust collectors and blast waves in dusty atmospheres. In addition, mixtures with gases heavily laden with particles occur frequently in industrial processes such as plastics manufacturing, flour milling, coal-dust conveying, powder metallurgy and powdered-food processing. General descriptions of such flows can be found in several books and review papers [Soo (1967), Marble (1970) and Rudinger (1973)].

The major differences between the flow fields which are developed behind a normal shock wave in a dusty-gas and a pure (dust-free) gas are illustrated in Figs. 1a and b for the temperatures and the velocities, respectively. When a steady pure gas encounters a normal shock wave it experiences a sharp (almost discontinuous) change in its thermodynamic and kinematic properties. This sudden change is shown in Fig. 1 to occur between (0) and (1). The thickness of this disturbance, l f, is only a few mean free paths of the gas atoms or molecules (about 6.6 x 10^-7 cm in standard conditions). Beyond (1) the gas properties remain constant (solid lines in Figs. 1a and b) provided the gas conditions at (1) are not sufficient to excite the internal degrees of freedom of the gas.

If, however, the gas is laden with solid particles then the suspension which was originally at a state of thermodynamic and kinematic equilibrium, ahead of the shock front, is sud-
suddenly changed into a non-equilibrium state, because the solid particles, due to their size compared with the initial disturbance, and the gas properties at (1) can be safely assumed to be identical to those of a pure gas with the same initial conditions.

For downstream of (1), i.e., at (∞) in Fig. 1, the gas and the solid phases reach a new state of thermodynamic and kinematic equilibrium via momentum and energy exchange. Theoretically all shock waves in dusty gases are infinitely thick, since equilibrium is approached asymptotically. However, it is a common practice to assign to the shock wave an effective thickness which is defined by a requirement that the suspension properties come close to their equilibrium downstream values. It was shown by Gottlieb and Coskunses (1985) that the suspension equilibrium properties (at infinity) can be calculated from the usual normal shock wave relations, provided that the usual pure gas parameters \( \tilde{\gamma} \) and \( \tilde{R} \) (the specific heat capacities ratio and the specific gas constant) are replaced by effective values \( \gamma \) and \( R \) which solely depend on the initial conditions of the suspension.

Between (1) and (∞) the solid particles are not in equilibrium with the gas. The flow region between (1) and ∞ is known as the relaxation zone, for it is analogous to the relaxation zone in pure gases where the internal degrees of freedom are excited. The extent of the relaxation zone strongly depends on the momentum and heat transfer mechanisms which enable the solid and the gaseous phases to reach a new equilibrium state. The analysis of the relaxation zone was studied by many investigators. The pioneering works of Carrier (1958), Kriebel (1964) and Rudinger (1984) verified the existence of this relaxation zone and identified the parameters affecting it, namely; the solid particle diameter, \( D \), its specific heat capacity, \( C_c \), its material density, \( \sigma \), and the loading ratio, \( \eta \). Igra and Ben-Dor (1989) compared various correlations for the drag coefficient, \( C_d \), and the heat transfer coefficient, \( Nu \), and pointed out their effect on the extent of the relaxation zone. In addition, they studied the role of thermal radiation heat transfer between the two phases and showed that it can be neglected when the incident shock waves Mach number is smaller than five.

In all the above-mentioned works, as well as in many others, the gaseous phase was assumed to behave as a perfect gas. This assumption was relaxed by Ben-Dor and Igra (1982) and Igra and Ben-Dor (1984) who solved the flow field while accounting for real gas effects. Dissociating nitrogen was the gaseous phase in the latter work and ionizing argon in the former.

The assumption that the solid particles are inert, which was also adopted in most of the published studies, was relaxed by Elperin, Ben-Dor and Igra (1986) who solved the flow field of an oxygen-carbon suspension passing through a normal shock wave, behind which the temperature of the carbon particles reached their ignition temperature and burned out.

The assumption of uniform solid particles was relaxed by Elata, Ben-Dor and Igra (1988) who solved the case of size-distributed solid particles.

In all the above-mentioned solutions, the flow field was one-dimensional and steady. The aim of the present study was to solve the case of a two-dimensional steady flow. This is the case when the shock wave is oblique. There are many incidences where the shock wave is oblique. For example, one can mention the shock wave generated by a supersonic vehicle, the shock wave which is developed at the entrance nozzle of a rocket engine and the reflected shock waves which arise when an explosion generated blast wave interacts with man-made structures. In all these cases the shock wave is oblique and, hence, unlike the previously mentioned cases the resulted flow field behind the shock wave is two-dimensional.

The aim of the present study, therefore, is to solve the flow field which is developed when a supersonic dusty gas suspension passes through a straight oblique shock wave.

Figure 2 illustrates schematically the problem to be solved. A dust gas suspension which is in a thermal and kinematic equilibrium encounters an oblique shock wave. The angle of incidence is \( \phi \) (sometimes known as the wave angle). As mentioned previously, upon the passage of the suspension through the shock wave, the properties of the gaseous phase assume a new state, known as the frozen state, immediately behind the shock front, while the solid phase passes through the shock wave unaffected. Due to the fact that the shock wave is oblique, the streamline of the gaseous phase is deflected by an angle, \( \theta_s \), while the trajectory of dust particles of the two phases which were mentioned previously, here there is also a difference in the direction of propagation of the two phases. The two phases which are no longer in equilibrium, start exchanging momentum and energy until they

\[ g(x,t) = \begin{cases} T_0 \quad \text{for } x \leq 0, \\ T_0 + \eta \frac{\gamma R}{\gamma - 1} \rho c^2 \quad \text{for } x > 0. \end{cases} \]