THE MECHANICAL EFFICIENCY AND KINEMATICS OF PANTOGRAPH-TYPE MANIPULATORS

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Pantograph mechanism has been well known for its motion feature of decoupled kinematics. Planar pantograph mechanism has been extensively used in machinery since the seventeenth century. Recently, three dimensional pantographs have been used in walking machine leg and manipulator designs. This is because, the pantograph mechanism possesses the following advantages: decoupled kinematics, higher energy efficiency, good rigidity, less link inertia and compact drive systems. In this paper, the mechanical efficiency of the kinematics of pantograph type manipulators are studied. The mechanical efficiency of pantograph mechanisms and conventional open-chain and closed-chain type manipulators are studied and evaluated using the concept of modified geometric work. The kinematics of six-d.o.f. pantograph type manipulators are studied and special mechanisms which simplify the kinematics are introduced. The computational complexity of both Cartesian and cylindrical type pantograph manipulators are evaluated and compared with a PUMA type manipulator.

Key Words: Pantograph Mechanism, Mechanical Efficiency, Inverse Position Analysis

1. INTRODUCTION

In a strict sense, the term pantograph mechanism is reserved for a special type of five-link mechanism (see Fig. 1) which possesses a decoupled kinematic relationship between the output motion of the reference point F and the two input degrees of motion. The closed-chain structure is a parallelogram and points A, B and F are maintained collinear at all times. The driving points of the two input degrees of freedom can be respectively assigned to points A and B, or they can be both assigned to either one of these two points. If the two input degrees of motion are driven separately by two linear actuators which are parallel with the two axes of the reference coordinate system, the kinematic relationship between input and output motion becomes decoupled. This type of pantograph mechanism, which was called simple pantograph in (Song, Lee and Waldrom, 1987) has been extensively used in embroidering machines, copying machines and magnifying mechanisms since the seventeenth century. Later, in the nineteenth century, a more general form of pantograph, the skew pantograph or the plagiograph, as it was called by its inventor, was introduced by Sylvester (Hobson, et al., 1953). Referring to Figure 2, the skew pantograph is obtained by attaching two similar triangulated rigid links to the parallelogram ACDE. If one carefully arranges the orientation of the two input linear axes with respect to the reference coordinate system, a decoupled kinematic relationship which is similar to that of the simple pantographs can be obtained. The way to define the orientation of the input linear axes was shown in (Song, Lee and Waldron, 1987) and will be reviewed in a later section.

Although both the simple and skew pantographs are planar mechanisms, they can be extended to a three-dimensional mechanism by the following two methods: The first method is to mount the frame on the base via a revolute joint (see Fig.

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Fig. 1 Simple pantographs

Fig. 2 A skew pantograph
Fig. 3 A cylindrical type pantograph manipulator

3. If the axis of the revolute joint is in the plane which contains the planar pantograph, the mechanism has decoupled kinematics in a cylindrical coordinate system. This type of pantograph is called a cylindrical pantograph. The second method is to mount the two input points A and B on a pair of revolute joints which have their axes parallel to one another (see Fig. 4). Thus, a lateral movement of one input point will cause the pantograph to stretch and rotate simultaneously, and the motion of the output point becomes three dimensional. Although a skew pantograph can generate three-dimensional motion, it was shown in (Song, Lee and Waldron, 1987) that only the simple pantographs have decoupled kinematics in Cartesian coordinate system. This type of three-dimensional pantograph is called Cartesian type pantograph and was first introduced by Hirose and Umetani in (Hirose and Umetani, 1980). They designed a quadrupedal walking machine with four Cartesian type pantograph legs.

In recent years, pantograph mechanisms have been frequently used in the robotics area, such as in the design of walking machine legs (Hirose and Umetani, 1980; Hirose, 1984; Hobson, et. al., 1953; Song, Waldron and Kinzel, 1985; Song and Lin, 1987; Yang and Lin, 1985), and manipulators (GAC Corp., 1982; Song and Lin, 1987; Yang and Lin, 1985). This is because, in addition to decoupled kinematics, this ancient mechanism possesses many other important advantages. These advantages include a high mechanical efficiency, high payload/weight ratio, low link inertia and compact drive systems. The high mechanical energy efficiency is because no geometric work is consumed during operation. This will be explained in detail in a later section. The high payload/weight ratio is due to the closed-chain structure of a pantograph. The low link inertia is because the actuators are mounted low on the rotating base. The compact size of the drive systems are due to the magnification motion feature of pantographs. Because of these advantages, pantograph mechanisms are especially suitable for the applications of walking machines, where computational efficiency of inverse kinematics and dynamics (Song, Lee and Waldron, 1987), energy efficiency and mechanical strength are important. Since the demands on computational and mechanical efficiencies of manipulators are becoming more strict, pantograph mechanisms will be more attractive in future manipulator design.

Although some of the advantages of pantograph mechanisms have been scatteredly mentioned in literature (Hirose and Umetani, 1980; Hirose, 1984; Hobson, et. al., 1953; Song, Waldron and Kinzel, 1985; Song and Lin, 1987; Yang and Lin, 1985), the advantages of high mechanical efficiency and decoupled kinematics were not fully investigated. Hence, it is the goal of this paper to present to the readers a complete study of these two aspects. In the following, the basic kinematics of Cartesian and cylindrical pantographs is reviewed first. The mechanical efficiency of the pantograph is then studied and compared with other types of manipulators. The kinematics of a six-d.o.f. pantograph type manipulators are is then studied in detail. Two wrist mechanisms which can simplify the kinematic relationships are also discussed.

2. BASIC KINEMATIC FEATURES OF PANTOGRAPHS

The basic kinematic features of pantographs are reviewed in this section. Figure 2 shows a skew pantograph mechanism. The angle \( \theta \) is called the skew angle. ACDE is a parallelogram and BCD and DEF are two similar triangulated links. It has been proven that \( \triangle AFB \) is similar to either \( \triangle BCD \) or \( \triangle DEF \) as long as ACDE is kept a parallelogram (Hobson, et. al., 1953). Thus, the motion of a pantograph can be fully represented by the imaginary, dashed triangle ABF, which was called equivalent triangle in (Song, Waldron and Kinzel, 1985). That is, given the positions of points A and B, the position of point F can be obtained by constructing the equivalent triangle.

The fundamental motion feature of a skew pantograph is stated as: If point A is fixed and point B traces a given curve, then point F traces a similar curve. The magnitude of the curve is magnified by a ratio \( R \) and the orientation is rotated through the skew angle \( \theta \) with respect to the given curve. Similarly, if point B is fixed and point A traces a given curve, then point F traces a given curve. The magnitude of the curve is magnified by a ratio \( R' \) and the orientation is rotated through an angle \( \phi \) with respect to the given curve, where

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R = \frac{AF}{AB}, \quad R' = \frac{BF}{AB} = \left(1 + R^2 - 2R \cos \theta \right)^{1/2}
\]

(1)