A General Probability Formula of the Number of Location Areas' Boundaries Crossed by a Mobile Between Two Successive Call Arrivals

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Abstract Mobility management is a challenging topic in mobile computing environment. Studying the situation of mobiles crossing the boundaries of location areas is significant for evaluating the costs and performances of various location management strategies. Hitherto, several formulae were derived to describe the probability of the number of location areas' boundaries crossed by a mobile. Some of them were widely used in analyzing the costs and performances of mobility management strategies. Utilizing the density evolution method of vector Markov processes, we propose a general probability formula of the number of location areas' boundaries crossed by a mobile between two successive calls. Fortunately, several widely-used formulae are special cases of the proposed formula.

Keywords mobile computing, location management, mobility management

1 Introduction

Personal Communication System (PCS) can provide wireless communication services to users on the move. To deliver services more effectively to a mobile user, it is important to have an efficient way to locate the mobile user[1,2]. Location management is used to track mobile subscribers in the PCS networks. Two prevailing existing standards for cellular technology, the Electronics Industry Association/Telecommunication Industry Association (EIA/TIA) Interim Standard 41 (IS-41), commonly used in North America, and the Global System for Mobile Communications (GSM), used in Europe and Asia, use a two-tier system of Home Location Register (HLR) and Visitor Location Register (VLR) databases to support location management. The basic architecture of PCS network is depicted in Fig.1, where MH, BS, MSC, and PSTN stand for Mobile Host, Base Station, Mobile Switch Center, and Public Switched Telephone Network respectively. The radio coverage of a BS is called a cell, and an LA (Location Area) consists of several cells and is managed by an MSC associated with a VLR.

PCS networks have experienced an explosive growth recently. To accommodate more subscribers, the size of cells must be reduced to make efficient use of limited frequency spectrum[3]. This, in turn, leads location management to a more difficult level. Therefore, location management is one of the most challenging topics in mobile computing. In the PCS networks with two-tier HLR/VLR databases, the system needs to update the current location information in order to track a mobile, whenever a mobile crosses the boundary of LAs. Hitherto, various location management schemes,
such as kinds of pointer forwarding schemes\(^4\)\(^-\)\(^9\) and others, have been proposed to reduce the cost of location management.

Therefore, studying the situation of a mobile roaming among the LAs is necessary and significant for conceiving and evaluating the mobility management schemes. To describe the number of LA’s boundaries crossed by a mobile, Lin\(^1\)\(^0\) derived a probability formula by means of probability theory, and Zhu\(^1\)\(^1\) obtained another probability formula by utilizing Vector Markov Process theory and Density Evolution Method\(^1\)\(^2\). However, the results of these two formulae are not the same since different starting points were chosen. This paper gives a general formula that includes and extends these two formulae.

This paper is organized as follows: in Section 2, we introduce a mathematical model of a mobile’s crossing LA’s boundaries between two successive calls, give the transition diagram of the system’s states, derive and solve the differential equations of the system’s states, and propose a theorem. In Section 3, we conclude the paper.

## 2 System Description

### 2.1 Assumptions

1) The incoming calls to a mobile form a Poisson process with rate \( \lambda \).

2) The location areas visited by the mobile are denoted by \( \text{LA}_0, \text{LA}_1, \text{LA}_2, \ldots \) in the order of visiting sequence.

3) The time of the mobile residing in \( \text{LA}_0 \) is a random variable \( X_0 \) with a general distribution \( H(x) \) and the density function \( h(x) \), i.e.,

\[
H(x) = \Pr\{X_0 \leq x\} = \int_0^x h(t)dt = 1 - e^{-\int_0^x \tau(t)dt}
\]

where \( \tau(t) \) is the failure rate function\(^1\)\(^3\), i.e.,

\[
\tau(t) = \frac{h(t)}{1 - H(t)}.
\]

Furthermore, suppose \( E[X_0] = \frac{1}{\tau} < \infty \), \( \text{Var}[X_0] < \infty \).

4) The time of the mobile residing in \( \text{LA}_n \) (\( n = 1, 2, \ldots \)) denoted by \( X_n \), is independent identically distributed random variables with a general distribution \( F(x) \) and the density function \( f(x) \).

i.e.,

\[
F(x) = \Pr\{X_n \leq x\} = \int_0^x f(t)dt = 1 - e^{-\int_0^x \mu(t)dt} \quad (n = 1, 2, \ldots)
\]

where \( \mu(t) \) is the failure rate function, i.e.,

\[
\mu(t) = \frac{f(t)}{1 - F(t)}.
\]

Furthermore, suppose \( E[X_n] = \frac{1}{\mu} < \infty \), \( \text{Var}[X_n] < \infty \).

5) All of the above random variables are mutually independent.

6) Whenever a mobile receives a call, the LA where the mobile is residing is numbered with 0, and its residence time is reset to 0.

### 2.2 Definition of the System’s States and Probability Density

We use \( \text{sign}(n) \) to indicate that a mobile is residing in \( \text{LA}_n \). Let \( X(t) \) be a mobile’s residence time and \( S(t) \) be the state of system at time \( t \). Thus, \( (S(t), X(t)) \) forms a vector Markov process\(^1\)\(^2\). The probability density of the system state at time \( t \), denoted by \( p_n(t) \), is defined as follows:

\[
p_n(t) = \Pr\{S(t) = (n), X(t) = x\} = \Pr\{S(t) = (n), x < X(t) \leq x + dx\}.
\]

Let \( p_n(t) = \Pr\{S(t) = (n)\} \). Obviously,

\[
p_n(t) = \int_0^\infty p_n(t, x)dx + \sum_{n=1}^\infty p_n(t) = 1.
\]

### 2.3 Transition Diagram of the System’s States

According to the above presuppositions, we can draw the following diagram to illustrate the transitions among the system’s states (see Fig.2).

![Fig.2. Transition diagram of the system’s states.](image-url)

### 2.4 Equations of the System’s States

Notice that the probability that a mobile is in \( \text{LA}_0 \) and its residence time is \( x + \Delta t \) (\( x > 0 \)) at time \( t + \Delta t \), is equal to the probability that the mobile is in \( \text{LA}_0 \) and its residence time is \( x \) at time \( t \), and besides, the mobile does not move onto another LA and there is no call arrival during \( \Delta t \). Therefore,

\[
p_0(t + \Delta t, x + \Delta t)dx = \Pr\{S(t + \Delta t) = (0), x + \Delta t < X(t + \Delta t) \leq x + \Delta t + dx\}
\]

\[
= \Pr\{S(t) = (0), x < X(t) \leq x + dx\}
\]