NUMERICAL ANALYSIS OF TURBULENT FLOW OVER A BACKWARD-FACING STEP USING REYNOLDS STRESS CLOSURE MODEL

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Reynolds stress turbulence models are adopted and applied for calculating turbulent flow over a backward-facing step. For the diffusion term in the transport equations for the Reynolds stresses, two gradient-type models are employed and compared. In addition, investigations on the modified $\varepsilon$ equations are carried out. The results of the computations are compared with the extant experimental data. As a consequence, it is concluded that the Reynolds stress models predict the flow field better than the standard $k-\varepsilon$ model in the recirculating region. However, after the reattachment the return to the ordinary turbulent boundary layer is shown to be too slow to predict the flow field irrespective of turbulence models.

Key Words: Reynolds Stress Closure Model, Modified $\varepsilon$ Equations, Backward-Facing Step

1. INTRODUCTION

Separation and reattachment of turbulent flow are important processes in many engineering applications, including diffusers, airfoils with separation bubbles, heat exchangers, and combustors. The turbulent flow over a backward-facing step is among the simplest that can display these processes. Although there have been many researches in this field, our current understanding of the reattachment process is still lacking because, despite the simplicity of the configuration, the flowfield is very complex, i.e., it consists of three zones such as recirculating region, reattaching region and redeveloping region.

Eaton and Johnston (1981) reviewed the general features of the backward-facing step flow. The separated shear layer appears to be much like an ordinary plane-mixing layer through the first half of the separated flow region. But the reattaching shear layer differs from the plane-mixing layer in the sense that the flow on the low-speed side of the shear layer is highly turbulent, as opposed to the low turbulence level stream in a typical plane-mixing layer experiment.

Bradshaw and Wong (1972) measured turbulence quantities in the reattachment and redeveloping regions. They indicated that shear stress and turbulence length scale in the reattachment region decrease spectacularly, mainly because of the confinement of the large eddies by the solid surface. Many researchers (Bradshaw and Wong, 1972; Kim et al., 1978; Chandrsuda and Bradshaw, 1981) reported the dip in the velocity profile near reattachment which persists for a downstream distance of about fifty step heights. The persistence of the dip implies that turbulence is not in local equilibrium and the length scale of the turbulence is not proportional to $y$, i.e., $l = ky$ but increases much more rapidly with $y$.

When flow separation occurs, turbulence structure becomes more complicated so that the theoretical study on turbulent separated flow becomes more difficult. Among the existing turbulence models, the Reynolds Stress Model, which provides transport equations for the Reynolds stresses, can explain anisotropic characteristics of turbulent diffusion while the standard $k-\varepsilon$ model cannot. This model has been systematically developed and improved by several researchers (Rotta, 1951; Daly and Harlow, 1970; Hanjalic and Launder, 1972; Launder et al., 1975).

In this study, the Reynolds stress closure model suggested by Launder et al. (1975) is applied to turbulent flow over a backward-facing step. To account for the fluctuating pressure field by the presence of the wall, the pressure-containing correlation model proposed by Gibson and Launder (1978) is also applied. The results are compared with the experimental data (Kim et al., 1978) and also partly with the standard $k-\varepsilon$ model.

2. GOVERNING EQUATIONS

The continuity and momentum equations describing the turbulent flow considered in the present study are of steady two-dimensional form as follows:

$$\frac{\partial}{\partial x_i} (\rho U_i) = 0, \quad (1)$$

$$\frac{\partial}{\partial x_i} (\rho U_i U_j) = - \frac{\partial \rho}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right] - \rho \frac{\partial U_i}{\partial x_j}, \quad (2)$$

2.1 Reynolds Stress Model(RSM)

We write the transport equations for the Reynolds stresses as

$$\frac{\partial}{\partial x_k} (U_k u_i u_j) = P_{ij} - \varepsilon_{ij} + \phi_{ij} + \phi_{ij,\omega} + D_{ij}, \quad (3)$$

where,
\[
P_{ij} = \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \text{ generation,}
\]
\[
\epsilon_{ij} = 2 \nu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \text{ dissipation,}
\]
\[
\phi_{ij} = \frac{\rho}{\partial x_j} \left( \frac{\partial u_i}{\partial x_i} + \frac{\partial u_j}{\partial x_j} \right) \text{ pressure-strain correlation,}
\]
\[
\phi_{ij,w} = \text{ pressure-strain correlation with near-wall correction,}
\]
\[
D_{ij} = -\frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} \right) \text{ diffusion.}
\]

Equation (5) is readily modelled by the well-known form given by Rotta (1951):

\[
\epsilon_{ij} = \frac{\rho}{3} \delta_{ij} \varepsilon.
\]

The pressure-strain term (6) is expressed by combining Rotta's linear return-to-isotropy hypothesis and the linear approximation of Launder et al. (1975):

\[
\phi_{ij} = -C_1 \frac{\varepsilon}{k} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \frac{\partial \delta_{ij}}{\partial x_j} P + C_2 \left( \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right),
\]

where \( C_1 = 1.5, C_2 = 0.4 \).

2.2 Transport Equation for Turbulence Energy Dissipation Rate

The transport equation for \( \varepsilon \) used in the high Reynolds number form of the \( k-\varepsilon \) model is given as

\[
\frac{\partial}{\partial x_j} \left( \rho u_i \frac{\partial}{\partial x_j} \varepsilon \right) = -C_1 \frac{\varepsilon}{k} (C_1 P - C_2 \varepsilon) + \frac{\partial}{\partial x_j} \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_j} \right)
\]

where the Boussinesq's eddy-viscosity concept is employed to express the production term \( P \) and \( \mu_t = C_\mu k^2/\varepsilon \) is the turbulent viscosity. In fact, Amano and Goel (1985) used this form of the transport equation for \( \varepsilon \) together with the transport equations for Reynolds stresses which are quite similar to those given in the previous subsection except the expression for \( \phi_{ij,w} \).

Instead, in the present study, we adopt the form of the transport equation for \( \varepsilon \) which was developed and used in Hanjalic and Launder (1972):

\[
\frac{\partial}{\partial x_j} \left( \rho u_i \frac{\partial}{\partial x_j} \varepsilon \right) = -C_1 \frac{\varepsilon}{k} (C_1 P - C_2 \varepsilon) + C_\varepsilon \frac{\partial}{\partial x_j} \left( \frac{k}{\varepsilon} \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} \right)
\]

The values of the constants used in the transport equations for \( \varepsilon \) are as follows:

\[ C_1 = 1.44, C_2 = 1.92, C_\varepsilon = 2.0, \sigma_\varepsilon = 1.22 \]

3. NUMERICAL METHOD

For the calculation of the flow field a modified version of the TEACH-2E code (Gosman and Ideriah, 1976) has been devised, which is compatible with the turbulence models described above and the necessary boundary conditions. The control volumes for mean-velocity components are the stag-