A Programmable Approach to Maintenance of a Finite Knowledge Base

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Abstract In this paper, we present a programmable method of revising a finite clause set. We first present a procedure whose formal parameters are a consistent clause set \( \Gamma \) and a clause \( A \) and whose output is a set of minimal subsets of \( \Gamma \) which are inconsistent with \( A \). The maximal consistent subsets can be generated from all minimal inconsistent subsets. We develop a prototype system based on the above procedure, and discuss the implementation of knowledge base maintenance. At last, we compare the approach presented in this paper with other related approaches. The main characteristic of the approach is that it can be implemented by a computer program.

Keywords knowledge base maintenance, propositional logic, predicate logic, clause set

1 Introduction

Knowledge base maintenance is an important topic in computer science. Many approaches to knowledge base maintenance have been introduced in recent years, among which the following approaches play an important role in knowledge base maintenance: truth maintenance system\(^1\), syntax-based approach\(^2,3\), model-based approach\(^4-9\), the logic of theory change\(^10\), and iterated maintenance\(^11-13\). Dixon\(^14,15\) first implemented the entrenchment-based maintenance of propositional base-generated approach\(^16\), then extended the method to predicate logic, and developed a prototype for knowledge base maintenance. Williams\(^17\) improved Dixon's maintenance strategy as follows: the entrenchment of a belief is not changed unless it is necessary. Damásio, Nejdl and Pereira\(^18\) introduced an approach to maintenance of the knowledge base of extended logic programming, and developed a prototype system called REVISE.

The implementation of knowledge base maintenance is a main topic in knowledge base maintenance area. Let \( \Gamma \) be a consistent belief set, and \( A \) be a belief. Many approaches described above define the revised belief set by the maximal subsets of \( \Gamma \) which are consistent with \( A \), but do not introduce an approach to yielding the maximal consistent subsets. We introduce an approach to generating all minimal subsets of \( \Gamma \) which are inconsistent with \( A \). Then we can gain all the maximal subsets of \( \Gamma \) which are consistent with \( A \). In this paper, a belief set is a finite clause set and a belief is a clause. So we use a clause set instead of a belief set and a clause instead of a belief. For the notation and terminology of logic and resolution, please refer to [19].

In Section 2, we present a procedure whose formal parameters are a clause set \( \Gamma \) and a clause \( A \) and whose output is in a set of minimal subsets of \( \Gamma \) which are inconsistent with \( A \), then we prove its correctness and discuss the implementation of knowledge base maintenance in Section 3. At last, we discuss the relationship between the approach presented in this paper and other related approaches. The main characteristic of the approach is that it can be implemented by a computer program.

2 A Procedure for Generating Minimal Inconsistent Subsets

In this section, we present a procedure to generate all minimal subsets of \( \Gamma \) which are inconsistent with \( A \) if \( \Gamma \) is inconsistent with \( A \), and prove its correctness.
correctness. We know that it is difficult to generate all minimally inconsistent subsets for a general first-order sentence set. But a first-order sentence can be transformed into a conjunction of clauses, so the rest of this section focuses on the clause form.

A procedure for generating all minimally inconsistent subsets is introduced in this section. Some lemmas are proved firstly.

**Lemma 2.1.** Let \( \Gamma \) be a clause set and \( A \) be the clause \( B_1 \lor B_2 \lor \ldots \lor B_n \). Suppose that \( \Psi_i \) is the set of all minimal subsets of \( \Gamma \) which are inconsistent with \( B_i \) (i = 1, 2, \ldots, n). \( \Psi \) is the set \( \{\Gamma_1 \cup \Gamma_2 \cup \ldots \cup \Gamma_n\} \) \( \Psi_i \in \Psi \), i = 1, 2, \ldots, n. Then the set \( \Psi \) contains all minimal subsets of \( \Gamma \) which are inconsistent with \( A \).

**Proof.** For each \( \Gamma' = \Gamma_1 \cup \Gamma_2 \cup \ldots \cup \Gamma_n \in \Psi \), because \( \Gamma_1 \vdash \neg B_1, \Gamma_2 \vdash \neg B_2 \land \ldots \land \neg B_n \), i.e., \( \Gamma' \vdash \neg A \). \( \Gamma' \) is inconsistent with \( A \). Let \( \Gamma' \) be the set \( \{\Gamma_1 \cup \Gamma_2 \cup \ldots \cup \Gamma_n\} \). Then \( \Psi \) contains all minimal subsets of \( \Gamma \) which are inconsistent with \( A \).

\( \square \)

\( \Psi \) may contain some clause sets which are inconsistent with \( A \), but not minimal. This is illustrated by the following example.

**Example 2.1.** Let \( \Gamma \) be \( \{A \lor \neg B, A \lor C, \neg C \lor \neg D, D, \neg A \lor \neg B\} \). \( \Gamma \) is inconsistent with the clause \( \neg A \lor B \). Let \( \Gamma_1 \) be \( \{A \lor C, \neg C \lor \neg D, D\} \). Then \( \Gamma_1 \) is a minimal subset of \( \Gamma \) which is inconsistent with \( \neg A \). Let \( \Psi_1 \) be \( \{\Gamma_1\} \). The minimal subsets of \( \Gamma \) which are inconsistent with \( B \) are the following two clause sets: \( \Gamma_2 = \{A \lor \neg B, \neg A \lor \neg B\} \) and \( \Gamma_3 = \{A \lor C, \neg C \lor \neg D, D, \neg A \lor \neg B\} \). Let \( \Psi_2 \) be \( \{\Gamma_2, \Gamma_3\} \), then \( \Psi = \{\Gamma_1 \cup \Gamma_2, \Gamma_1 \cup \Gamma_3\} \) contains all minimal subsets of \( \Gamma \) which are inconsistent with \( \neg A \lor B \). The subset \( \Gamma_1 \cup \Gamma_2 \) of \( \Gamma \) is inconsistent with \( \neg A \lor B \), but not minimal, so the set \( \Gamma_1 \cup \Gamma_2 \) can be removed from \( \Psi \) and \( \Psi \) is used to denote the set \( \{\Gamma_1 \cup \Gamma_2\} \). Then \( \Psi \) is the set of all the subsets of \( \Gamma \) which are minimally inconsistent with \( \neg A \lor B \).

By Lemma 2.1, \( \Psi \) is the set of all minimal inconsistent subsets if the clause sets which are a subset in \( \Psi \) are removed from it. And the set of all minimal subsets of \( \Gamma \) which are inconsistent with \( A \) can be generated from the sets \( \Psi_i \), where \( \Psi_i \) is the set of all minimal subsets of \( \Gamma \) which are minimally inconsistent with \( B_i \) (i = 1, 2, \ldots, n).

**Lemma 2.2.** For a clause set \( \Gamma \) and literal \( B \), let \( \Delta \) be the set \( \{C | C \text{ is a resolvent of } B \text{ and a clause in } \Gamma\} \). \( \Gamma' \) be the set \( \{C | C \in \Gamma, \text{ and } B \text{ does not imply } C, \text{ and the resolution operator cannot be applied to } B \text{ and } C\} \). For each resolvent \( C_i \in \Delta \), let \( \Psi_i \) be the set of all minimal subsets of \( \Gamma' \cup \Delta - \{C_i\} \) which are inconsistent with \( C_i \). Let \( \Psi' \) be \( \bigcup_{i=1}^{n} \Psi_i \), and \( \Psi \) be the set \( \{\Gamma'' - \Delta \cup \{D | E \in \Gamma'' - \Delta \text{ is the resolvent of } D \text{ and } E\} | \Gamma'' \in \Psi'\} \), then \( \Psi \) contains all minimally consistent subsets of \( \Gamma \) which are inconsistent with \( A \).

**Proof.** For each \( \Gamma'' \in \Psi_i \), there exists a resolution refutation for \( \Gamma'' \) and \( C_i \), because \( C_i \) is minimal inconsistent with \( \Gamma'' \). If the clauses in \( \Delta \) used in the deduction of resolution refutation are substituted by its parent clause in \( \Gamma \), then a resolution refutation for the clauses in \( \Gamma \) and \( B \) is obtained, because each clause in \( \Delta \) is a resolvent of \( B \) and a clause in \( \Gamma \). And the clauses used in the deduction of resolution refutation are clause \( B \) and the clauses in the following set: \( \Gamma_1 = \Gamma'' - \Delta \cup \{D | E \in \Gamma'' - \Delta \text{ is the resolvent of } D \text{ and } E\} \). We know that \( \Gamma_1 \) is a consistent set because \( \Gamma'' \) is a consistent set. But \( \Gamma_1 \) is inconsistent with \( B \). Suppose that \( \Gamma_1 \) is not minimal. Let \( \Gamma' \) be a proper minimal subset of \( \Gamma_1 \) inconsistent with \( B \). For a clause \( C \) in \( \Gamma' \), if the resolution operator can be applied to \( C \) and \( B \), then \( C \) in \( \Gamma' \) is substituted by the resolvent of \( C \) and \( B \). The new clause set is denoted by \( \Gamma_k \). Then \( \Gamma_k \), a proper subset of \( \Gamma'' \), is inconsistent. This contradicts with the fact that \( \Gamma'' \) is minimally inconsistent with \( B \). Hence \( \Gamma_1 \) is minimally inconsistent with \( B \).

Secondly, we prove that \( \Psi \) contains all minimal subsets of \( \Gamma \) which are inconsistent with \( B \). For a minimal subset \( \Gamma' \) of \( \Gamma \) which is inconsistent with \( A \), let \( \Delta \) be the set \( \{D | D \text{ is the resolvent of } B \text{ and a clause in } \Gamma'' \} \). \( \Gamma'' \) be the set \( \Delta \cup \{D | D \in \Gamma'' \} \), and the resolution operator cannot be applied to \( D \) and \( B \). Then \( \Gamma'' \) is minimally inconsistent. Furthermore, for each \( C \in \Delta \), \( \Gamma'' - \{C\} \) belongs to \( \Psi_i \), and \( \Gamma' = \Gamma'' - \Delta \cup \{D | B \in \Gamma'' - \Delta \text{ is the resolvent of } D \text{ and } C\} \). Hence, each minimal subset of \( \Gamma' \) which is inconsistent with \( B \) belongs to \( \Psi \).

From the above discussion, we know that the theorem holds. 

For a clause set \( \Gamma \) and the clause \( A = B \lor B_2 \lor \ldots \lor B_n \), by Lemmas 2.1 and 2.2, in order to construct minimal subsets of \( \Gamma \) which are inconsistent with \( A \), all minimal subsets of \( \Gamma \) which are inconsistent with \( B_i \) need to be generated for each \( B_i \) (i = 1, 2, \ldots, n). Furthermore, in order to con-