VARIANT MODIFICATION OF THREE-PARAMETER
K~MA DISTRIBUTION

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Fifty years ago S. N. Kritskii and M. F. Menkel proposed an original probability distribution which they called
three-parameter gamma distribution [2, 3, 5, 6]. It became widespread in hydrotechnical construction — when constructi-
s and bridges, in water supply under the condition of using surface waters — and in most scientific works in geophysi-
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For structural designing it is preferable to map the parameters of the probability distribution being used, therefore mapping of the parameters of the three-parameter gamma distribution \( C_v \) and \( C_s/C_v \) is now used widely calculating river runoff.

It is suggested to consider several blocks under the probabilistic model:

- analysis of the physical essence of the investigated characteristic and formulation of a hypothesis of variability of characteristic;
- formation of a representative information sample;
- analysis of the theoretical properties of certain a priori suitable analytical probability distributions;
- selection of several optimal cumulative distribution functions in accordance with the proposed principle of multivariate geophysical designing [13];
- construction of analytical and empirical cumulative distribution functions on a. Hazen’s probability paper;
- determination of the correspondence of each analytical CDF to the empirical CDF by means of statistical goodness-of-fit tests or by another method;
- analysis of analytical quantiles and selection of the design one.

The proposed calculation algorithm differs substantially from the algorithm recommended by the building codes S 01.14-83 [9].

In addition to Hazen’s known probability paper, which gained wide use in domestic and foreign scientific practical hydrology, some other types of so-called rectifying graph paper, on which a particular CDF is a straight line, known. In our opinion the arguments advanced when creating a number of such probability papers differ little from arguments when creating Hazen’s probability paper; hence all other probability papers are only analogues (with no fundamental differences) of Hazen’s probability paper. Furthermore, the abundance of probability papers hinders the task of optimal and standardization of planning and surveying works.

The probability distribution function of the three-parameter gamma distribution has the form [2, 3, 5]:

\[
\rho(x) = \frac{1}{\Gamma(\gamma) b^\gamma} \left( \frac{x}{b} \right)^{\gamma - 1} \frac{\Gamma(\gamma + b)}{\Gamma(\gamma)} x^{\gamma/b - 1} \exp \left\{ \frac{\Gamma(\gamma + b) x}{\Gamma(\gamma)} \right\}^{1/b},
\]

here \( \bar{x} \) is the arithmetic mean of the distribution; \( \gamma, b \) are distribution parameters and also are related to coefficient \( C_v \).

The distribution has a lower zero limit for any nonnegative values of \( x \).

The variability of various hydrologic characteristics is formalized in the probability density function by variation in the interval \([0, +\infty]\). Such a hypothesis is universal and mathematically completely correct. However, its realization constant zero lower limit of the probability distribution is not the only possible one [8]. G. G. Svanidze emphasized that any hydrologic quantities there are a real physical maximum and a real nonzero physical minimum.

Consequently, if for a statistical probability analysis of maximum runoff the presence of a rigid lower limit equal to zero can be allowed under the condition of a good approximation of extremely large values of small exceedance probabilities (the left tail of the empirical distribution), then when analyzing the minimum runoff the given feature of the three-parameter gamma distribution leads to understatement of the design minimum discharges in those cases when the real investigated se is a finite nonzero physical minimum.

Consequently, it is necessary to modify the right tail (or region of large exceedance probabilities) of the three-parameter distribution for an adequate probabilistic investigation of extremely small values.

Theoretically, two methods exist for this. The first is the introduction of the location parameter into the probability density function and its determination by means of a class of invariant statistics of the standard deviation, coefficient of skewness, etc., type. However, this leads to a change in the density function and in essence to a completely new distribution of the form

\[ f(x) = \frac{1}{\Gamma(\gamma) b^\gamma} \left( \frac{x - \mu}{b} \right)^{\gamma - 1} \frac{\Gamma(\gamma + b)}{\Gamma(\gamma)} \left( \frac{x - \mu}{b} \right)^{\gamma/b - 1} \exp \left\{ \frac{\Gamma(\gamma + b) (x - \mu)}{\Gamma(\gamma)} \right\}^{1/b}, \]

The term "statistic" is used by us for denoting both the probability distribution parameter and nonparametric statistic, with independent significance and use in various statistical-geographic problems and generalizations.