Weyl’s Gauge Field and Its Equation with Meron Solution (*)

M. NISHIOKA

Department of Physics, Faculty of Liberal Arts
Yamaguchi University - Yamaguchi 753, Japan

(ricevuto il 26 Agosto 1983)

Summary. — We study Weyl’s gauge field interacting with a scalar field which is self-interacting. For a special self-interacting term of the scalar field, we obtain an equation composed of only Weyl’s gauge field. By combining the equation obtained with the condition in Weyl’s geometry that the length scale of any vector changes under parallel transfer, we obtain a nonlinear equation for the length scale of Weyl’s gauge field (vector), which is found to have meron solution. This solution is compared with $SU_2$ Yang-Mills field.

PACS. 11.10. – Field theory.

1. – Introduction.

The geometrical structure of Weyl’s manifold (1) is determined by the following two geometrical concepts: a) an affine connection which is defined by the change due to infinitesimal parallel transfer of a vector $\xi^\mu$ from a point $P(x^\mu)$ to $P'(x^\mu + dx^\mu)$,

$$
\delta \xi^\mu = - \Gamma^\mu_{\nu\lambda} \xi^\nu dx^\lambda,
$$

(1)

where $\Gamma^\mu_{\nu\lambda} = \Gamma^\mu_{\lambda\nu}$ (coefficients of symmetric connection), b) the metric tensor $g_{\mu\nu}$

(*) To speed up publication, the author of this paper has agreed to not receive the proofs for correction.

which is defined by the length scale $\xi$ of $\xi^\mu$,

$$\xi^2 = g_{\mu \nu} \xi^\mu \xi^\nu,$$

and the condition that the length scale changes under parallel transfer

$$\delta(\xi^2) = \delta(g_{\mu \nu} \xi^\mu \xi^\nu) = -f \xi^2 A_\mu \mathcal{A}^\mu,$$

where $A_\mu$ is Weyl's gauge field, $f = \text{const}$.

In spite of its beautiful formulation, Weyl's theory of a so-called unified model was not accepted by physicists. UTIYAMA (2) introduced a new scalar field called a measure field to avoid the two defects of Weyl's theory of gauge field which were: i) an invariant distance $ds$ at any world point located in a gauge field cannot have a definite magnitude, ii) the Einstein equation in the conventional form violates the gauge invariance. By measuring all the quantities with a standard given by the measure field, it is shown by UTIYAMA that all the field equations can be written in a manifestly gauge-invariant manner. An arrangement of Utiyama's theory with scalar self-interacting term was given by the present author (3), which derived an equation composed of Weyl's gauge field only. In sect. 2 we summarize results (2,3) for later use. In sect. 3 we will combine the equation obtained in sect. 2 with the condition in Weyl's geometry that the length scale of any vector changes under parallel transfer, and obtain a nonlinear equation for the length scale of Weyl's gauge field (vector).

Next we study solutions for the nonlinear equation and find a meron solution for it. Moreover, we make remarks on the two-dimensional solution. Finally, in sect. 5 we compare the solution obtained with the $SU_2$ Yang-Mills gauge field.

2. - Formulation.

Following UTIYAMA (2), we consider the gauge-invariant and generally covariant Lagrangian density $L_\varphi$ for the measure field $\varphi$

$$L_\varphi = -\sqrt{-g} \left( \frac{a}{2} g^{\mu \nu} \nabla_\mu \varphi \nabla_\nu \varphi \right),$$

where $\nabla_\mu = \partial_\mu - \frac{1}{2} f A_\mu$, $A_\mu$ is Weyl's gauge field, $f$ is the coupling constant and $a$ is a constant.