On a Functional Approach to the Lee Model (*)

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Summary. — Schwinger's functional differentiation methods are employed to approach the Lee model from a relativistic point of view. Topics discussed include: commutation relations and indefinite metric, sum rules, and spectral representations. It is argued that Heisenberg's results using a Hilbert space with indefinite metric are not in contradiction with the functional description based on the action principle. The technical advantages of the functional methods are indicated by an extended Lee model which allows for pair creation.

1. — Introduction.

The content of this paper is meant to be an improved treatment of the problems encountered in the Lee model (L.M.). We present a rather detailed description of the dynamics of this familiar model (1) using functional techniques rather than the more standard Hilbert-space methods (1,2). These techniques were first given by FRIED (3) and are further developed here. The main idea

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(2) S. S. Schweber: An Introduction to Relativistic Quantum Field Theory (1961).

in this context is to extend the nonrelativistic, nonlocal theory into its relativistic realm; then, transcribe its local nontrivial generalization into functional formulation, employ techniques which are largely known from field-theoretical calculations, and finally regain the L.M. by certain approximations performed upon the correct causal Green's functions which enter the theory. These techniques are especially suited to an understanding of the solubility structure of a fairly simple model and, beyond this, are a most striking way of establishing an approach to more complex models as, e.g., Heisenberg’s nonlinear spinor theory as displayed in ref. (4).

The special dynamics of the L.M. is governed by the relatively simple particle propagators involved. The V-particle propagator has a self-energy structure that is generated by iterating the most simple proper self-energy graph which is of second order in the coupling constant, while the N- and 0-particle carry no structure at all. The model becomes soluble in the sense that one can construct in terms of mass and charge renormalization a completely finite theory, without the need of a cut-off function. However, the eigenvalue equation for the renormalized mass of the V-particle needs some special care. Here Heisenberg’s regularization procedure will provide us with a tool best suited to overcome the difficulties which arise from the nonlocality of the original L.M. Various modifications of Lee’s original results can be studied to great advantage from the viewpoint of functional methods. One extension is facilitated by considering the case in which the 0-particle carries the self-energy structure. The analysis in which the last model is combined with the L.M. has also been carried through. The Appendix will supply the reader with a number of Green’s functions used in the text.

2. — Construction of the generating functional (5,6).

Consider the Lagrangian describing coupled relativistic V, N and 0-particle fields, i.e.

\[ \mathcal{L} = \mathcal{L}_0 + \mathcal{L}', \]

where \( \mathcal{L}_0 \) refers to the sum of the free-particle Lagrangians, e.g.

\[ \mathcal{L}_0^{(\nu)} = - \bar{\psi}_\nu (m_\nu + \gamma \cdot \vec{c}) \psi_\nu, \]

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(6) K. Symanzik: e.g. in Lectures at the Summer School for High-Energy Physics at Hercegovi (Yugoslavia, 1961).