John Pell and His Correspondence with Sir Charles Cavendish

by Noel Malcolm and Jacqueline Stedall

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The obvious question on taking up this 650-page book is why anyone would be interested in the life, mathematics, and letters of John Pell. Pell has always been a somewhat mysterious character, mentioned in passing in various accounts of English mathematics of the seventeenth century. Most mathematicians and historians of mathematics would be hard pressed to name any mathematical result or, indeed, any mathematical idea that Pell contributed, except that Leonhard Euler, having come across a discussion of the equation \( x^2 + Ny^2 = 1 \), for integers \( N \), \( x \), and \( y \), in John Wallis's *Treatise of Algebra*, erroneously named it in Pell's honor. (The only relationship of Pell to this equation seems to be that he took William Brouncker's solution and rewrote it in his own three-column method.) Yet after reading the detailed biography of Pell by Noel Malcolm and the analysis of his mathematics by Jacqueline Stedall, which precede the edition of the letters between Pell and Cavendish, an important point becomes clear: the progress of mathematics does not depend only on the "geniuses" who create the mathematical ideas, but also on those who study their works and communicate them widely.

As Malcolm informs us, many of the biographical details of Pell's life in standard reference works are erroneous, so he went to considerable trouble to verify and correct the details, citing numerous references for virtually every important item. Thus, we learn that Pell was born on 1 March 1611, that he matriculated at Trinity College, Cambridge in the Easter term of 1624 (at the precocious age of 13), that he received his BA in 1629, and that shortly thereafter he took a position as a school teacher. Pell married Ithamara Reginalds in 1632; they ultimately had eight children. Sometime in 1629, Pell made the acquaintance of Samuel Hartlib, in whose circle Pell developed his long-term interest in questions of pedagogy and, in fact, not only in reforming entirely the teaching in the schools but also in the reformation and perfection of all human knowledge. It was during the 1630s that he first made grandiose plans to publish, but he never finished the work. For example, as he wrote in 1638, "no one, so far as I know, has dared to attempt this task, namely, to register in a catalogue all the simple sounds of human language, and all the simple concepts of the mind, and to express the complications of the latter by means of the combinations of the former. May God... allow me to begin, complete, and publish that work, for the glory of his name and the unspeakable benefit of the human race." But this project, like so many others he conceived, never came to fruition.

It was also in the 1630s that Pell studied mathematics seriously, including the works of William Oughtred, Thomas Harriot, Bartholomew Pitiscus, Albert Girard, and François Viète. As he studied these works, he made notes for himself and planned to organize these into a new and improved text on algebra, especially on "this whole new doctrine of Equations." Over the years, his friends and acquaintances begged him to complete and publish such a work, but, as usual, he did not accommodate them. In addition, Pell busied himself with producing tables, primarily long tables of logarithms. In 1634, he made plans to produce a book on logarithms and trigonometry, including "the Logarithms of every number under 100 thousand and of 2, 3, 5, 7, 9, Sines, Tangents and Secants for every Degree and Cen- esin of degrees... to a Radius of 10,000,000,000 particles." But though his manuscripts contain probably thousands of pages of tables, no such book ever appeared.

Pell was also interested in various questions on optics, and it was through that interest that he was introduced to Sir Charles Cavendish, who would become, during the 1640s, Pell's patron and avid supporter. Cavendish had an interest in the design of telescopes, and, as his early correspondence with Pell demonstrates, asked for assistance in designing and manufacturing the best lenses for this purpose. But evidently, he also employed Pell to teach him mathematics, and constantly awaited a book from Pell which would encompass the new algebra. Thus, as early as 1641, Cavendish would write, "I confess I expect not an exact book of analytics i.e., algebra till you perfect yours." This sentiment would be a continuing theme of Cavendish's letters over the next decade—since Pell would never, in fact, "perfect" his algebra.

Despite his lack of publications, however, Pell was well-regarded in mathematical circles. Thus it was in 1644 that, with the help of friends, he secured an appointment to the Chair of Mathematics at the Athenaeum in Amsterdam, what one might today call a "preparatory school" for the higher university faculties of law, medicine, and theology. Two years later, he moved on to another similar institution in the Netherlands, the Illustre School te Breda, where he remained until 1652. Most of his remaining years were spent in England, with the exception of a two-year diplomatic stint in Zurich. And with the restoration of the monarchy in 1660, Pell was able to secure a Church living, as a rector in Essex. He also became one of the earliest members of the Royal Society. And it was in 1668 that virtually the only mathematical work written by Pell appeared in print, his comments on and additions to the English edition of Johann Rahn's *Teutsche Algebra*. Unfortunately, although Pell found patrons during the last years of his life, much of that time he was in very straightened circumstances, in part because he had a son-in-law and grandson who were spendthrifts.

During his long life Pell, who died in 1685, was always regarded as a mathematician. For many years he was employed as a teacher of mathematics, but what contributions did Pell make to mathematical knowledge? As Stedall remarks, "for all the prodigious time and effort Pell expended on mathematics, one is left with an overriding sense of incompleteness." As she notes, Pell left thousands of pages of assorted notes on mathematics, most of which have been,
randomly bound into over thirty volumes that are now held in the British Library in London. However, his publications are very few and even his assorted notes do not show evidence of great mathematical creativity. Nevertheless, he did contribute to mathematics.

One major part of Pell's mathematical opus deals with algebra. He had read and absorbed the work of Viète, Harriot, Oughtred, and Descartes, among others, and attempted to develop their work further and systematize it. In fact, he planned to design an "algebra of knowledge," in which complex ideas could be built from simple notions: "If we had all our simple notions set downe, we had as perfectly all our simple sounds, we have perfectly all y' words & speeches of men potentially." But such an algebra was clearly not possible in practice.

What Pell did accomplish was a method of attacking algebraic problems in a standardized way, his method of three columns. In this method, the middle column simply contains the line numbers. The left-hand column begins with the list of unknown quantities and then continues with instructions on manipulating the various lines to get new lines. Finally, the right-hand column begins with the known relationships, written in algebraic notation, and then following the instructions on the left, gives the various changes in the relationships until, at the end, the solutions to the unknowns appear. Among Pell's own notational innovations were the division sign (+) and the use of lower case letters for unknown quantities, replaced by capital letters as soon as the numerical solution was found. For example, if in a particular problem, lines 7, 8, and 9 were $4a - b - c = 400$, $-a + 3b - c = 300$, and $-a - b + 2c = 200$, respectively, then with the instruction for line 10 reading "$7 + 8 + 9$," line 10 itself reads $2a + b = 900$. And, again in this problem, the final line, 19, reads $c = 5300/13$. In general, Pell followed Harriot in his algebraic notation, using juxtaposition for multiplication and just using repetitions of a letter for powers, rather than exponents. But he did represent "equal" by our modern sign (=) rather than by Harriot's own symbol.

While Pell was in Zurich in the mid 1650s, he taught his methods to Johann Heinrich Rahn, who in 1659 published his Teutsche Algebra, clearly based on Pell's tutoring and displaying the three-column method. Several years later, the book was translated into English by Thomas Branker, who then asked Pell to make additions and improvements. Pell acceded to this request and, although it took several years to complete, the new version appeared in 1668. This book contains some of the basics of the theory of equations, including the results that an equation has as many roots as the highest power and that the second highest power of an equation can always be removed by appropriate substitution. But the major part of the book consists of problems with solutions, both "arithmetical and geometrical," most of which Pell added to Rahn's work. The first problem is, given any two of the quantities $a + b$, $a - b$, $ab$, $aa + bb$, $aa - bb$, to find the remaining four.

One other work that Pell published (1647), after spending what seems an inordinate amount of time on the project, was a brief book in which he refuted the work of C. S. Longomontanus, who claimed that the ratio of the circumference of a circle to its diameter was $\pi = 18,252: 43$. Pell refuted this by using repeated bisection to find the length of a circumscribed polygon of 256 sides, applying a new double-angle formula for the tangent to aid in the calculation, a formula equivalent to the modern

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}.$$  

In fact, Pell used the formula backwards, starting with the tan 45° = 1 and then finding the tangent of the half angle until, after six bisections, he arrived at the tangent of 0°42 3/16'. He was certainly correct in his refutation, but he delayed the publication until he could get several mathematicians to endorse his methods. In fact, some of these mathematicians gave a proof of the double-angle formula, whereas Pell himself did not produce one.

In his correspondence with other mathematicians, Pell solved numerous other problems, mostly algebraic or trigonometric, and evidently a good bit of what he knew made its way into John Wallis's Treatise of Algebra. But since Pell often promised to write up pieces of work, "when he had leisure" and did not do so, it is difficult to know exactly what he had in mind. In addition, although Pell was certainly current in the algebraic developments of the first half of the seventeenth century, the work in analysis of Fermat, Descartes, Roberval, and ultimately Newton had little effect on him. Nevertheless, he maintained his reputation as a mathematician, not because of any major contribution, but because he had a general understanding of at least a part of the mathematics of his time that was better than that of most of his contemporaries.

The second half of the book under review, the complete correspondence between Pell and Cavendish, provides a fascinating view into the intellectual lives of two-seventeenth century personalities. There is only a little mathematics in these letters, mostly Pell explaining some ideas to Cavendish or Cavendish writing up something that he does not quite understand. But we also learn a good deal about how intellectual work was accomplished before easy communication was available. Each correspondent comments on the work of others; each reports on who he had visited with recently, and when he would expect a new work of another to be published. In their letters, they like each other know whose book has appeared when, suggesting frequently that the other try to find a copy locally.

Given that Pell was certainly the better mathematician of the two, it is comforting to see that he is very gentle in his mathematical criticisms of Cavendish. He tells Cavendish when his proofs are circular and suggests ways of improving them. He asks Cavendish to attempt to solve problems and then either compliments him on a correct solution, or gives him hints as to how to improve the solution. And he patiently explains to Cavendish how the method of analysis works and why one needs to invert the analysis argument to get a correct synthetic proof. But the theme that most pervades these letters, particularly the more mathematical ones, is Cavendish's constant plea to Pell to "publish his analyticks" so that the world will be able to learn from him. Pell always replies that he does not have the leisure to do this.