A plane-polar near-field to far-field transformation with the fast Hankel transform method

Chen CAO*
Emmanuel VAN LIL*
Antoine VAN DE CAPELLE*
Kees VAN 'T KLOOSTER**

Abstract

It is known that a direct radial integration, used to compute the far-field from uniformly spaced plane-polar near-field measurements requires the evaluation of a large amount of Bessel functions and hence CPU time. Up to 1985 only unequally spaced fast Hankel algorithms were available. Hansen [3] developed an algorithm that was usable for equally spaced measurements points, but only for order zero. His theory is generalised in this paper and applied to a plane-polar near-field to far-field transformation.

Key words: Champ électromagnétique, Champ proche, Champ lointain, Coordonnée polaire, Transformation Hankel, Algorithme rapide, Analyse numérique.

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I. INTRODUCTION

The transformation from plane-polar near-field to far-field is known to require a double integration for every far-field point. The first part is an azimuthal integration including trigonometric functions, that can be evaluated by using FFT. The second part is the radial integration which includes Bessel functions. The latter is complicated to evaluate and hence inefficient for computational purposes. Due to this radial integration, the efficiency of a plane-polar near-field to far-field transformation is much lower than the efficiency of plane-rectangular near-field to far-field transformation which requires only a 2-dimensional fast Fourier integration.
In order to raise the efficiency of a plane-polar near-field to far-field transformation, a new computational way for the evaluation of the Hankel transform with uniform sampling points is developed [1], [2]. This method substitutes the Hankel transform by an Abel transform followed by a Fourier transform. The Abel transform is interpreted as a linear shift-variant system. With the basic theory of a linear shift-variant system, the Abel transform can be evaluated recursively. We can summarize this method in the following steps:

2. Application of the theory of linear shift-variant systems to calculate the Abel transform [4].
4. Analytical integration of the Abel transform, assuming a linear behaviour of the near-field function.

We note that no Bessel function calculation is required any more.

In this paper, the method is developed in the first section. In the second section the computation of the impulse response in the shift-variant system with two numerical methods, namely Prony [8], and Levenberg-Marquardt (L-M) [7], is approximated. In the third section, the fast Hankel method is tested on a simulated near-field. The antenna under test is a $16 \times 16$ array of Hertzian dipoles, located in the $x-y$ plane, directed along the $y$-axis, and spaced $0.5 \times 0.5$ wavelengths. The far-field pattern has a $-24$ dB Chebyshev distribution in the $x-z$ plane and a $-30$ dB Chebyshev distribution in the $y-z$ plane.

II. DEVELOPMENT OF THE METHOD

The conventions used for the measurement are shown in figure 1a, and figure 1b, where the antenna under test is located in the $x-y$ plane, $r$ is the distance from the antenna's phase center to an observation point, $\theta$ and $\phi$ are the elevation and azimuth angle respectively. The measured near-field disk is located in the $x'-y'$ plane where the $\phi'$ is the azimuthal angle and $a_0$ is the radius of the near-field measurement disk (see Fig. 1b).

Starting with an expansion in the $\phi$-direction, we obtain:

\[ E_t(s', \phi') = \sum_{n=0}^{N} \left( A^n(s') \cos n\phi' + B^n(s') \sin n\phi' \right) \]

where $E_t$ is the tangential near-field. The coefficients are the Fourier transform of the near-field with respect to $\phi'$:

\[ \begin{align*}
A^n(s') &= \frac{\epsilon_n}{2\pi} \int_{0}^{2\pi} E_t(\phi', s') \cos n\phi' \, d\phi' \\
B^n(s') &= \frac{\epsilon_n}{2\pi} \int_{0}^{2\pi} E_t(\phi', s') \sin n\phi' \, d\phi'
\end{align*} \]

with

\[ \epsilon_n = \begin{cases} 
1 & \text{for } n = 0, \\
2 & \text{for } n \neq 0.
\end{cases} \]

The far-field can be written as [1]:

\[ \overrightarrow{E}_f(\theta, \phi) = C_0 jk_0 \cos \theta e^{jk_0(d \cos \theta - r)} \sum_{n=0}^{N} \frac{1^n(\overrightarrow{H}_z^n \cos n\phi + \overrightarrow{H}_z^n \sin n\phi)}{r} \]

where $A^n(s')$ and $B^n(s')$ are the Fourier coefficients of the near-field. The far-field pattern is calculated from the Fourier coefficients with a linear shift-variant system.