Ionospheric wave spectrum measurements *

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Abstract

A significant quantity to be measured in the ionosphere is the local spectrum, $S(k, \omega)$, of either potential or electron density fluctuations. One can determine from $S(k, \omega)$ characteristics of the macroscopic plasma such as the local density and temperature, transport coefficients, and drift current. Determination of $S(k, \omega)$ may be carried out by measurement of the cross-power spectrum. Signals from two probes are passed through a cross-power spectrum analyzer, and the measurement is repeated for a series of probe separations. The resulting cross-power spectra, $H(r, \omega)$, must be Fourier-transformed to obtain $S(k, \omega)$. A number of questions pertinent to the use of this method in space are discussed. These include the expected signal-to-noise ratio, and the frequency and wave-number resolution attainable.

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A significant quantity to be measured in the ionosphere is the local spectrum, $S(k, \omega)$, of either potential or electron density fluctuations. For the latter, one can determine from $S(k, \omega)$ characteristics of the macroscopic plasma such as the local density and temperature, transport coefficients, and drift current. The measurement of $S(k, \omega)$ in the ionosphere has normally been carried out by the incoherent scatter technique, using ground-based radars. Recently, there has been discussion of using this same technique with radars located on the Space Shuttle, but analyses have shown this to be unpromising [1].

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There is available for determining \( S(k, \omega) \) another technique, based upon a passive in situ measurement [2]. In this method, the signals from two probes are passed through a cross-power spectrum analyzer, and the measurement is repeated for a series of probe separations, \( r \). The cross-power spectra are Fourier-transformed with the aid of a computer to obtain \( S(k, \omega) \). We have successfully used this technique in the laboratory on current-driven ion-acoustic turbulence [3-5].

In this paper, we shall examine the practicality of using the cross-power spectrum analyzer on the Space Shuttle to measure ionospheric parameters. In particular, we shall investigate the integration time required to make a measurement of the cross-power spectral density to a given accuracy.

2. THEORY

The cross-power spectrum analyzer is shown schematically in figure 1. Signals from two probes immersed in the ionosphere pass through a filter bank, where the frequency band is selected. The signals are then multiplied and averaged, to give the real part of the cross-power spectral density, \( H(r, \omega) \) [2]. By shifting the signal in one of the arms by \( 90^\circ \), and then multiplying and averaging, one obtains the imaginary part of \( H(r, \omega) \). Alternatively, one could sweep the frequency by replacing the filter banks by a sweep oscillator, twin mixers, and a pair of fixed filters.

In our theory, we assume that the probes are biased into electron saturation. The reasons for this are, first, that our measurements on current-driven ion-acoustic turbulence were found to be most successful with the wave-detecting probes biased into electron saturation, and second, the sheath thickness, and therefore the effective size of the probe, is smallest under these conditions.

We shall study two cases which we believe are of general interest: (2.1.) the ion line in an equilibrium plasma, and (2.2.) the plasma line in an equilibrium plasma.

2.1. Electron density fluctuations.

The starting point for our theory is the mean-square fluctuation of the electron density per unit bandwidth as a function of \( S(k, \omega) \) [2],

\[
H(0, \omega) = \frac{1}{(2\pi)^2} \int S(k, \omega) \, dk.
\]

To determine this quantity, we must first know \( S(k, \omega) \).

For a quiescent plasma, the formula for \( S(k, \omega) \) is given simply by [6]

\[
S(k, \omega) = \frac{2 n(k\lambda_D)^2}{\omega} \left( \frac{1}{1 + \chi_e^2} \right) \left( 1 + \chi_i^2 \right) \left( 1 + \frac{\omega}{\omega_i} \right)
\]

where \( \chi_e \) and \( \chi_i \) are the electron and ion contributions to the susceptibility, and \( \omega \) denotes the imaginary part. Setting \( \omega = 0 \) in this formula gives a representative value for the ion-line,

\[
S(k, \omega) = \frac{2 n(k\lambda_D)^2}{\omega} \left( \frac{1}{1 + \chi_e^2} + \frac{\omega}{\omega_i} \right).
\]

Substituting (4) into (2), and integrating yields the mean-square fluctuation per unit bandwidth. For the two cases, these are found to be given by

\[
\begin{align*}
(5a) & \quad H(0, \omega) = \frac{n}{(2\pi)^3} \omega \left( k_{\lambda_D}^2 \right) \left[ 2 + (k\lambda_D)^2 \right] \\
(5b) & \quad H(0, \omega) = \frac{n}{(2\pi)^3} \omega \left( k_{\lambda_D}^2 \right) \left[ 2 + (k\lambda_D)^2 \right]
\end{align*}
\]

These results were obtained by integrating (2) over a sphere in \( k\)-space of radius \( k_m \). By the Nyquist sampling theorem [7], this quantity is given by

\[
(6) \quad k_m = \pi h,
\]

where \( h \) is the sampling increment for the probe separation, \( r \).

The bandwidths corresponding to the wavenumbers, \( k_m \), are easily shown to be given for the two cases by

\[
\begin{align*}
(7a) & \quad \Delta f = k_{\lambda_D} \left( 2\pi \right)^{1/2} \\
(7b) & \quad \Delta f = v_i \left[ \left[ 1 + 3 (k\lambda_D)^2 \right]^{1/2} - 1 \right]
\end{align*}
\]

where \( v_i \) is the ion thermal velocity.