Brannen and Ferguson\textsuperscript{1} have reported experimental results which they believe to be incompatible with the observation by Hanbury Brown and Twiss\textsuperscript{2} of correlation in the fluctuations of two photoelectric currents evoked by coherent beams of light. Brannen and Ferguson suggest that the existence of such a correlation would call for a revision of quantum theory. It is the purpose of this communication to show that the results of the two investigations are not in conflict, the upper limit set by Brannen and Ferguson being in fact vastly greater than the effect to be expected under the conditions of their experiment. Moreover, the Brown-Twiss effect, far from requiring a revision of quantum mechanics, is an instructive illustration of its elementary principles. There is nothing in the argument below that is not implicit in the discussion of Brown and Twiss, but perhaps I may clarify matters by taking a different approach.

Consider first an experiment which is simpler in concept than either of those that have been performed, but which contains the essence of the problem. Let one beam of light fall on one photo-multiplier, and examine the statistical fluctuations in the counting-rate. Let the source be nearly monochromatic and arrange the optics so that, as in the experiments already mentioned, the difference in the length of the two light-paths from a point $A$ in the photocathode to two points $B$ and $C$ in the source remains constant, to within a small fraction of a wavelength, as $A$ is moved over the photocathode surface. (This difference need not be small, nor need the path-lengths themselves remain constant.) Now it will be found, even with the steadiest source possible, that the fluctuations in the counting-rate are slightly greater than one would expect in a random sequence of independent events occurring at the same average rate. There is a tendency for the counts...
to ‘clump’. From the quantum point of view this is not surprising. It is typical of fluctuations in a system of bosons. I shall show presently that this extra fluctuation in the single-channel rate necessarily implies the cross-correlation found by Brown and Twiss. But first I propose to examine its origin and calculate its magnitude.

Let $P$ denote the square of the electric field in the light at the cathode surface in one polarization, averaged over a few cycles. $P$ is substantially constant over the photocathode at any instant, but as time goes on it fluctuates in a manner determined by the spectrum of the disturbance, that is, by the ‘line shape’. Supposing that the light contains frequencies around $v_0$, we describe the line shape by the normalized spectral density $g(v - v_0)$. The width of the distribution $g$, whether it be set by circumstances in the source itself or by a filter, determines the rate at which $P$ fluctuates. For our purpose, the stochastic behaviour of $P$ can be described by the correlation function $P(t)P(t + \tau)$, which is related in turn to $g(v - v_0)$ by

$$\overline{P(t)P(t+\tau)} = \overline{P}^2(1 + |\rho|^2),$$

where \(\rho = \int_{-\infty}^{\infty} g(x) \exp 2\pi i x \, dx \quad (1)\)

For the probability that a photoelectron will be ejected in time $dt$, we must write $\alpha P dt$, where $\alpha$ is constant throughout the experiment. It makes no difference whether we think of $P$ as the square of an electric field-strength or as a photon probability density. (In this connection the experiment of Forrester, Gudmundsen and Johnson on the photoelectric mixing of incoherent light is interesting.) Assuming one polarization only, and one count for every photoelectron, we look at the number of counts $n_T$ in a fixed interval $T$, and at the fluctuations in $n_T$ over a sequence of such intervals. From the above relations, the following is readily derived: