CROSSED ORBITS IN THE RESTRICTED PROBLEM OF THREE BODIES WITH REPULSIVE AND ATTRACTIVE FORCES.

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§ 1.

Introduction.

The problem here considered is to determine periodic orbits for two infinitesimal bodies which are attracted by a finite body but repelled by each other, the forces of attraction and repulsion being according to the Newtonian law of the inverse square. The three bodies in this problem are analogous to the electrons and the nucleus of the helium atom and for this reason the infinitesimal bodies will be called «electrons» and the finite body the «nucleus».

A particular solution of the problem is that in which the electrons revolve in circles with the nucleus as centre and remain diametrically opposite. These circles lie in the same plane, hereafter considered as the «equatorial plane». Two types of orbits are obtained when the electrons are displaced from the circular motion. In Part I the electrons remain diametrically opposite and equidistant from the nucleus, that is, their «longitudes» differ by 180°, their «latitudes» are the same in magnitude but opposite in sign or direction. In Part II the distances of the electrons from the nucleus are equal, their longitudes differ by 180° but their latitudes are the same. The particular case which is common to Part I and Part II, viz., that in which the latitudes are zero, is discussed in Part II.

The results which have been obtained are similar to those which were first proposed by Kémble and later independently by Bohr for the «crossed orbit» model.
of the normal helium atom, considered from the standpoint of the quantum theory \(^1\)). In the present paper only the classical mechanics is used.

The orbits in Part II (that is those which are not in the equatorial plane) are also similar to those which may be obtained by rotating the arc-orbits found by Rawles \(^2\) about the axis of symmetry which does not intersect the orbits.

The orbits obtained are approximately similar to the spatial curves formed by "wrapping a sine curve about a barrel-shaped figure \(^3\)" which varies only slightly from a figure of revolution.

There would be only a slight change in the treatment in Part I if the electrons were considered to be of finite mass.

\[ \text{§ 2.} \]

**The Differential Equation.**

A rectangular system of axes is taken having the origin at the nucleus and the coordinates of the electrons are \(x_1, y_1, z_1\) and \(x_2, y_2, z_2\). Let the ratio of the repulsion to the attraction be denoted by \(k\) and let the units of time and of space be chosen so that the gravitational constant of attraction is unity. The force function of the system is then

\[
U = \frac{1}{r_1} + \frac{1}{r_2} - \frac{k^2}{r_{12}},
\]

where

\[
\begin{align*}
    r_1 &= (x_1^2 + y_1^2 + z_1^2)^{\frac{1}{2}}, \\
    r_2 &= (x_2^2 + y_2^2 + z_2^2)^{\frac{1}{2}}, \\
    r_{12} &= [(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2]^{\frac{1}{2}},
\end{align*}
\]


\(^3\) Van Vleck, op. cit., p. 90.