Mathematics of the Gregorian Calendar

V. Frederick Rickey

October 15, 1982 was the beginning of the second 400 year cycle of the Gregorian calendar, an event insuffi-
ciently celebrated by mathematicians. Although a
number of recent articles deal nicely with the history
of the calendar, one has to go back to early editions of
the Encyclopaedia Britannica [12]—a place few would
think to look—to find a good treatment of the math-
ematical issues involved. While most of these are ele-
mentary, the calendar does give rise to many curious
facts, some of which are presented in italics below as
exercises (with references for the lazy).

By 45 B.C., the old Roman calendar was in such a
chaotic state, due mostly to political manipulations,
that Julius Caesar, with the advice of the Graeco-Egyp-
tian astronomer, Sosigenes, declared that every year
divisible by four was to be a leap year with 366 days;
the other years were to be common years of 365 days.
The extra day was inserted before the sixth day before
the Calends of March, hence that month contained
two sixths, or was, as we still say, bissextile. The new
year then began in March, thus explaining why the
months September through December are etymologi-
cally the seventh through tenth.

In the Julian calendar, the years had an average
length of 365.25 days while the tropical year was then
11 minutes 4 seconds shorter and so every 131 years
the calendar retrogressed one day with respect to the
seasons. This defect was known at the time of reform,
but it took several centuries for the calendar to become
out of whack enough to matter.

Scores of scholars devised schemes for revising the
Julian calendar. Regiomontanus (1436–1476), the fa-
ther of trigonometry as a science independent of astron-
omy, became involved at papal behest, but before
he could do anything he was poisoned by the sons of
a man that he had written a polemic against. This was
doubly unfortunate: the Protestant reformation was
yet to come and change would have been easier. In
1514 Pope Leo X asked Copernicus for assistance, but
he declined saying that the length of the year was not
known precisely enough; this was one of his reasons
for writing De Revolutionibus.

Pope Gregory VIII appointed a commission of math-
ematicians, astronomers, and clerics to revise the cal-
endar. It was led by the mathematician, Christopher
Clavius (1537–1612) who is generally given most of the
credit for the reform, primarily since he defended it so
staunchly. The plan that was used was the work of
the physician, Luigi Lilio, who died shortly before.
(For additional historical details, see Moyer [7] or
Philip [8].) Gregory’s reform, which was promulgated
in the papal bull Inter gravissimas (Of the gravest con-
cern) of February 24, 1582, was intended to solve sev-
eral problems:

1. How could the year be brought back in accord
   with the seasons?
2. How could the Julian leap year rule be corrected
   so as to stop further dislocation?
3. Where should the beginning of the year be fixed?
4. How should the date of Easter be determined?

Although the new leap year rule now seems like the
important point, it was actually the Easter issue that
motivated papal intervention. The determination of
the date of Easter has always been of mathematical
interest, but it is complicated enough to deserve a note
of its own.

V. Frederick Rickey
In A.D. 325 the Council of Nicaea "fixed" the vernal equinox at March 21, but by the sixteenth century it had slipped ten days behind. To convince Pope Gregory XIII of this the Tower of Winds was built at the Vatican containing a Meridian Room with an inlaid marble meridional sundial line across the floor. At noon on March 11, 1582, not March 21 as it should have, the sun shone in through the mouth of the South Wind, a mural on one wall, and crossed the meridional line.

This room had another connection with mathematics. In 1655, when Christina converted to Catholicism and abdicated as Queen of Sweden, she lived in this tower. She was, you will remember, the spartan queen who contributed to Descartes' death by insisting that he tutor her in an unheated library early in the morning, contrary to his custom of staying abed until noon. For years there were rumors that parts of the frescoes had been overpainted before she arrived, perhaps by adding drapery to the figures so as not to offend her. Later when the paintings were restored it was learned that what was overpainted was a paraphrase of the Biblical motto Jeremiah 1:14, "All bad things come from the North." No doubt this would have offended her.

Bringing the calendar back into accord with the seasons was accomplished by declaring that the day after Tuesday, October 4, should be Wednesday, October 15, in the year 1582. These dates were chosen so as to minimize the importance of the feast days omitted. Dropping these ten dates brought the equinox back to the time of the Council of Nicaea (A.D. 325), not to the time of Caesar's reform, thereby giving the whole thing an ecclesiastical touch. Nonetheless, this retrospective aspect was a mistake; it has confused historians ever since.

The reform was adopted quickly by Catholic Europe, but it was resisted elsewhere both for religious and academic reasons. For example, François Viète (1540–1603) condemned the reform as a corruption of the Julian calendar. Certainly the two bitterest critics were Michael Maestlin, Kepler's teacher, and Joseph Justus Scaliger. (Scaliger's father made his reputation by attacking Cardano. When Cardano didn't reply immediately Scaliger thought he had died; Scaliger had a change of heart and wrote a laudatory funeral oration. Then Cardano replied.) Although "the calendar reform literature is on the whole 'interesting to few and entertaining to none' and a scholar of sense and taste will readily turn to other labours rather than cultivate this barren field" [11], the history of this dispute has been interestingly presented by Moyer [7].

The calendar was not adopted quickly. It was still possible in 1908 for a University of Michigan professor to state that he traveled for 43 days in August in Northern Europe, but that his September had only eighteen days. The explanation here is the same as why the "October Revolution" is celebrated in November—Russia did not adopt the Gregorian calendar until 1918.

Shakespeare and Cervantes died on the same date. Which one died first?

The New Leap Year Rule

The Gregorian reform also introduced a new leap year rule which was designed to keep the equinox near March 21. In the Gregorian calendar every year divisible by four is a leap year, except that years divisible by 100 are common, except years divisible by 400 are leap. Thus the Gregorian calendar has 97 leap years every 400 years, three fewer than the Julian. For example, the year 1500 was a common year under the new Gregorian calendar, but not under the Julian. Thus England, which did not adopt the reform until 1752, fell one day further behind. Judging by past performance, the year 2000 will confuse many. It will be a leap year, but it will not be the first year of the 21st century, rather the last of 20th.

The adoption of the Gregorian calendar was more traumatic in England than elsewhere. John Wallis, the theologian appointed as the first Savilian Professor with no more mathematical reputation than breaking a few coded messages for the king, strongly opposed the new calendar as a papish plot.

The delay in England led to additional historical confusion. George Washington's birth was recorded as 11 February 1731/32 O.S. This is an old style or Julian date. It became 22 February in our calendar. The "slash date" indicates that the year was 1731 in England, but 1732 on the continent. This is because the beginning of the year was then celebrated on March 25 in England. It was also the day taxes were due. When the Gregorian change occurred in 1752, the beginning of the year was changed to January 1 (this was also part of the Gregorian reform), but the tax date was delayed eleven days to April 5, where it remains. Apparently the tax collectors couldn't do fractions.

The 400 year cycle of our calendar leads to a number of peculiar results. My favorite—for it never fails to perplex—is that the thirteenth of the month is more likely to occur on Friday than on any other day of the week. B. H. Brown [1] posed this as a Monthly problem in 1933 and it continues to appear [5]. Since there are $400 \times 12 = 4800$ thirtieths of the month in one cycle and 7 does not divide 4800, it is clear that the 13th does not occur equally often on the different days of the week. (This is an example of a non-constructive existence proof that was not done by contradiction, something many feel can't happen.) To show that Friday is the most likely day for the 13th to occur on requires a brute force compilation with a perpetual cal-