Some Thoughts on the Teaching of Mathematics

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I recently saw a play about a young Welshman who becomes a history teacher in an English public school. He begins his first lesson by asking a student which side—King or parliament—was victorious in England’s first civil war. After a few more technicalities comes this: “Trevelyan thinks England’s first civil war was the most important single event in English history. What do you think?” This teacher and I share an educational philosophy that views the critical and evaluative function of education as its central contribution to the intellectual and spiritual development of the student. Uninterpreted or underinterpreted technical material—the all-too-common fare of generations of students in elementary school, high school, and college—destroys mind and soul, and teaches cynicism and contempt for teacher and subject alike.

No subject is feared and despised as much as mathematics, for it is both difficult and apparently meaningless. And yet this very subject can be used to quantify symmetry and continuity, to supply the tools for the most sophisticated physical theories, to disclose the richness and bounds of form and even the limitations of deductive reasoning itself.

Why this prodigious gap between image and reality? Why “math anxiety?” I have no doubt that the root cause of this deplorable and depressing state of affairs is the all-too-prevalent elimination from the teaching of mathematics—from kindergarten through college—of the critical and evaluative components and the concentration on the teaching of mathematical “facts” and “skills.” All too often what we teach is locally rational but globally meaningless and incoherent, a Golem without memory, perspective or soul.

And then there is the manner in which we minister to our students. All too often this manner is an expression of a stifling paternalism that makes us endlessly prescribe tasks to the point where the word “initiative” itself disappears from the student’s vocabulary.

It is not my intention to glorify students and denigrate teachers. What I am concerned with here is not how many students want to learn but how many teachers can teach, that is, how many teachers have the resources to induce some, however few, students to cooperate in their intellectual and human growth. And since all teachers are trained and certified by the universities, it is ultimately the universities that must become the initiators of change.

The key to significant teaching of all students on all levels is the technical and intellectual competence of their teachers. The need for technical competence on the part of the teacher is obvious. The need for critical and historical competence of the teacher, as distinct from his technical competence, is less appreciated. This justifies my preoccupation with the intellectual aspect of mathematical education.

Many elementary and high school teachers have
poor technical mathematical training and virtually no training in the history and philosophy of mathematics. They see themselves as purveyors of a variety of mathematical technology but certainly not as part-time critics and intellectual historians.

Many university teachers regard the production of new results as a quintessential good and view students who do not show research potential, or at least extremely high technical competence, as a kind of "educational ballast" needed for the perpetuation of an extensive educational enterprise. Told that they should function not only as purveyors of mathematical technology but also as part-time critics and intellectual historians, many university teachers would be just as surprised as their colleagues in elementary and high schools.

If all this is substantially correct, then the required repair job is obvious. The universities must see to it that the haphazard technical training of elementary and high school teachers is replaced by thoughtful training that minimizes technical difficulties while emphasizing mathematical issues. Beyond this, the universities must see to it that all categories of teachers are made aware of the historical and critical dimensions of mathematics. Finally, and most importantly, the universities must see to it that all categories of teachers are aware that the ultimate educational objective is to produce free, that is, autonomous, men and women with a love of, or at least a regard for, learning. So much for my blueprint for change.

It is no secret that the actual teaching of mathematics is seldom an experiment in significant communication. More often than not it is a process in which a "superior" indoctrinates "inferiors." There is no gain in student autonomy and no cooperative critical monitoring of the enterprise by teacher and students. What work is done is decreed rather than done voluntarily in an effort to reach deeper understanding. One down-to-earth manifestation of the low psychological quality of the educational process, of its paralyzing paternalism and prescriptiveness, is that few students of mathematics below the graduate level acquire the habit of consulting, on their own, a book other than a textbook.

Am I naive enough to think that my blueprint for change is a magic wand that would banish all these evils and usher in the educational millennium? Most emphatically not. But the blueprint is a good one; training teachers along the indicated lines is a fundamental step in the right direction.

I am fully aware that while we are called teachers we are put in the position of doubling as regulators of social traffic, as gatekeepers. What is tragic is that many of us are so caught up in the latter aspect of our role that we tend to forget about the teaching role and its primary and ultimate objective spelled out above. This primary goal must never be lost sight of. In practical terms, this means that we must take advantage of every opportunity to develop our students' taste and teach them to teach themselves. The critical habit to be instilled is the habit of critical reading.

A final comment. We would like our students to be good at mathematics. But what if they are not? Must they then end up feeling diminished rather than augmented by our ministrations? I do not think so.

I make it absolutely clear to my students that in spite of my enthusiasm for mathematics I regard mathematical skill as just that—a skill. I tell them that what I put first in a human being is his humanity. I tell them that when I see a person confront a difficulty, in particular a mathematical difficulty, what I admire most is not aptitude but perseverance and stubborn patience. These qualities are ours to employ.

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It remains for me to exemplify what I mean by teaching intellectually significant mathematics to averagely endowed audiences. I wish to preface the description of my examples with a "programmatic" remark.

A necessary (though certainly not sufficient) condition for significant teaching is the provision of emphases; if everything is important, then nothing is important. Some teachers emphasize logical completeness, others applications, still others genetic factors and so on. I regard the genetic approach as central to the discussion of any topic in mathematics. What matters is the tracing of the historical evolution of an idea or concept and the pursuit of what one might call comparative mathematics. No matter how modest the scope of a presentation, the lecturer must place the issues discussed in a historical and intellectual context and discuss their significance. Now the examples.

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**Example I**

**A topics course in mathematics [1]**

What follows is a brief description of a course I introduced into a program offered by the department of