EFFECT OF COHERENCE IN PASSAGE OF LIGHT
THROUGH A THIN LAYER

A. P. Khapalyuk and N. A. Klyukanova

The coefficients of transmission and reflection of light by a plane-parallel plate depend greatly on its thickness and the wavelength [1, 2]. In the literature a distinction is usually made between thick and thin layers. A study of the transmission of light through thick layers does not require a consideration of its wave properties and can be made within the framework of the theory of geometric optics. Thin layers show interference effects and in the consideration of these in the existing literature it is assumed that the waves are monochromatic, i.e., the characteristic coherence length of the beam incident on the layer is infinite. The two theories (geometric and interference) lead, of course, to different results. There is no exact criterion, however, of their applicability. In addition, there are intermediate cases where interference needs only partial consideration. The essential factors are not only the thickness of the layer and the wavelength, but also the coherent properties of the light. This paper is devoted to the consideration of such an intermediate case. The aim was to find out how the coefficients of reflection and transmission of light through a plane-parallel layer depend on the degree of coherence of the beam and the thickness of the layer (or wavelength) and to determine the criteria of applicability of the two theories mentioned above.

1. We consider a homogeneous isotropic nonabsorbing plane-parallel plate of infinite extent, of thickness 2l, and with refractive index $n_2$. Let the $z$ axis of a Cartesian system of coordinates be directed perpendicular to the surface of the plate. The refractive indices of the media bounding the layer are denoted by $n_1$ and $n_3$, respectively. The light wave incident normally on the layer is characterized by a prescribed angular frequency $\omega_0$ and coherence time $(2T)$ or length $(L = cT)$. According to Maxwell's equations such light can be regarded as a set of trains (pulses) of plane monochromatic waves of a particular length. If we regard the individual wave pulses as independent of one another (incoherent), we can confine ourselves to a consideration of only one of them. We write such a pulse of a plane-polarized wave with unit amplitude in the form

$$E_i^1 = e^{i(k_0 (ct - n_1 z) - L, L)}, \quad H_i^1 = n_1 E_i^1, \quad (1)$$

where $h(x; a, b)$ - a characteristic function of the interval - is given by the equation

$$h(x; a, b) = \begin{cases} 1 & a < x < b \\ 0 & a > x, x > b. \end{cases} \quad (2)$$

The spectral composition of the pulse (1) can easily be found by means of a Fourier integral

$$F_i(\omega) = \int_{-\infty}^{\infty} E_i^1 (ct - n_1 z) e^{-ik_0 (ct - n_1 z)} d\omega = 2 \sin L (k - k_0) \frac{\sin T (\omega - \omega_0)}{\omega - \omega_0}. \quad (3)$$

Hence, it follows that the pulse (1) can be regarded as the integral (sum) of plane monochromatic waves with amplitudes corresponding to formula (3). The spectral component of this integral is written in the form

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Fig. 2. Reflection coefficient of layer as function of parameter $4k_0n_2l$ for two values of Fresnel reflection coefficient on surface of layer and different degrees of coherence of the beam. Continuous lines for $r = 0.08$, broken lines for $r = 0.04$. 1) $p = \infty$ (geometric optics); 2) $p = 4$; 3) $p = q = 2$; 4) $q = 4$; 5) $q = \infty$ (interference optics).

Here we ignore dispersion, which corresponds approximately to the case where the medium has no absorption frequencies close to $\omega_0$.

To find the wave reflected and transmitted by the layer we can first find the spectral component (4) reflected and transmitted by the layer, and then integrate with respect to $\omega$. The spectral component (4) is a strictly monochromatic wave and, hence, in the treatment of its reflection and transmission by a layer of any thickness we must give full consideration to interference effects. The reflection and transmission coefficients of such a wave are well known [2]. The spectral component of the reflected ($E_{1\omega}^b$) and transmitted ($E_{3\omega}^a$) wave can be written in the form

$$E_{1\omega}^b = -\frac{R_{21} + R_{32}e^{-i\omega_0l}}{1 + R_{21}R_{32}e^{-i\omega_0l}} F_1(\omega) e^{ik(ct-n_2z)} ,$$

$$E_{3\omega}^a = \frac{(1 - R_{33})(1 - R_{33})e^{i2\omega_0l}}{1 + R_{33}R_{32}e^{i2\omega_0l}} F_1(\omega) e^{ik(ct-n_2z)} ,$$

where the Fresnel reflection coefficients $R_{21}$ and $R_{32}$ at the boundaries of the layer are given by the formulas

$$R_{21} = \frac{n_2 - n_1}{n_2 + n_1}, \quad R_{32} = \frac{n_3 - n_2}{n_3 + n_1}.$$

The reflected and refracted waves of the incident pulse (1) are given by the integrals

$$E_1^b = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{R_{21} + R_{32}e^{-i\omega_0l}}{1 + R_{21}R_{32}e^{-i\omega_0l}} \sin L(k - k_0) \ e^{ik(ct+n_2z)} \ dk,$$

$$E_3^a = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{(1 - R_{33})(1 - R_{33})e^{i2\omega_0l}}{1 + R_{33}R_{32}e^{i2\omega_0l}} \sin L(k - k_0) \ e^{ik(ct-n_2z)} \ dk.$$

Integrals (6) can easily be calculated by using an expansion in a series ($R = -R_{21}R_{32}$)

$$\frac{1}{1 - R_{21}R_{32}} = \sum_{s=0}^{\infty} R_{21}R_{32}e^{-i\omega_0l}$$

and changing the order of summation and integration. The reflected and refracted waves are written in the form

$$E_1^b = -\left[ R_{21}h(ct+n_2z; -L, L) - \frac{1}{\pi} \sum_{s=0}^{\infty} R_{21}R_{32}h(ct+n_2z-4sn_2d; -L, L) \right] e^{ik(ct-n_2z)},$$

$$E_3^a = \left[ (1 - R_{33}) \sum_{s=0}^{\infty} R_{32}h(ct+n_2z; -L, L) - 2n_2d; -L, L) \right] e^{ik(ct-n_2z)}.$$

These waves consist of pulses of the same length as the incident pulse but with gradually diminishing amplitudes. The time taken to pass a prescribed point in space (point $z_1$) is different: They will follow one another at constant intervals $\tau = 4n_2d/c$, which depend only on the optical thickness of the layer. If we consider the passage of a pulse in a prescribed time the partial pulses will be arranged in space (along the $z$ axis) one after the other with a uniform displacement $\Delta z = c\tau = 4n_2d$. Thus, formulas (8) correspond to the known concept of the passage of light through thin layers as multiple reflection from the two boundaries of the layer [3].

Depending on the relationship between the length of the pulse (or the coherence length $L$) and the optical thickness of the layer the partial pulses either do not overlap at all, or partially overlap. Complete overlap of the pulses only occurs in the limiting cases: when the thickness of the layer is reduced to zero (no layer) or $L$ is increased to infinity (strictly monochromatic wave). Despite the fact that all the partial