Rhapsody in White: A Victory for Mathematics

In 1999, the Breckenridge International Snow Sculpture Championships saw its first mathematical surface: the Costa surface, whose production in snow was reported on in [2]. That effort might have set the stage, for this year another minimal surface took several awards at the same event.

Robert Longhurst has used the ideas of negative curvature in much of his sculpting work in wood, and his piece showing an Enneper surface (Figure 1) seemed ideal for realization in the hard snow that Breckenridge prepares. The team, again sponsored by Wolfram Research, Inc. (makers of Mathematica), consisted of Longhurst, a wood and stone sculptor from Chestertown, New York; Dan Schwalbe and Stan Wagon, who had learned the rudiments of sculpting negative curvature from Helaman Ferguson at the 1999 event; Andy Cantrell, a sophomore at Macalester College; and John Bruning of the Tropel Corporation, the nonsculpting photographer for the team. It was through Bruning that the rest of the team was introduced to Longhurst’s work.

We sculpted an Enneper surface of degree 2 (see [1]), a minimal surface that was discovered by A. Enneper in 1864.
Inspired by the swooping curves, we named it "Rhapsody in White." Unlike the Costa surface, Enneper's surface does not embed in 3-space (it has self-intersections) and is topologically dull (it is homeomorphic to the plane). But the process of truncating the infinite surface just before the self-intersections yields a surface that, at least from a sculptural perspective, has a very beautiful shape. One must drill and then expand several holes, which gives a topological flavor to the work. The openness of the result yields quite a pleasant view. And the negative curvature that occurs at each point gives the piece structural strength that allows it to be built out of snow.

The parametric representation [1] is simple:

\[ f(r, \theta) = \left( r \cos \theta - \frac{1}{5} r^5 \cos(5\theta), \quad r \sin \theta + \frac{1}{5} r^5 \sin(5\theta), \quad \frac{2}{3} r^3 \cos(3\theta) \right) \]

Plotting \( f \) with \( r \) varying from 0 to 1.4 and \( \theta \) in \( [0, 2\pi] \) leads to Figure 1, though in that figure the \( x \)-axis is the vertical axis and the viewpoint is on the positive \( z \)-axis. Figures 2 and 3 show two views of our sculpture.

There were 17 teams at the January, 2000, event, from England, The Netherlands, Germany, Switzerland, Finland, Russian, Mexico, Canada, and the U.S.; for images of almost all of the pieces see [3]. The audience’s reaction told us that our work had succeeded and had the desired impact. The curves were swooping and graceful, the overhang exciting, and the surface smooth. But how would the art judges react to a purely mathematical shape? Would its entrancing form win them over, or would they find it unimaginative? When third place was announced to the Swiss team’s punctured sphere, we became concerned, for we thought that the judges would not award two medals to geometric shapes. But then the silver medal was awarded to us. First place went to a soaring Russian structure illustrating human striving. The judging seemed fair; but the next night, we became convinced that our work had thoroughly won over all viewers, when we received both the People’s Choice award (voting by the approxi-