

Inverse Entailment and Progol

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Abstract This paper firstly provides a re-appraisal of the development of techniques for inverting deduction, secondly introduces Mode-Directed Inverse Entailment (MDIE) as a generalisation and enhancement of previous approaches and thirdly describes an implementation of MDIE in the Progol system. Progol is implemented in C and available by anonymous ftp. The re-assessment of previous techniques in terms of inverse implication leads to new results for learning from positive data and inverting implication between pairs of clauses.

Keywords: Learning, Logic Programming, Induction, Predicate Invention, Inverse Resolution, Inverse Entailment, Information Compression.

§1 Introduction

Since its inception in this journal³¹⁾ Inductive Logic Programming (ILP) has grown to become a substantial sub-area of both Machine Learning and Logic Programming (see Ref. 43)). The success of the subject lies partly in the choice of the core representation language of logic programs. Least Herbrand models of logic programs²⁶⁾ fit neatly with the distinction between examples and conjectured theories in inductive inference. The syntax of logic programs provides modular blocks which, when added or removed, generalise or specialise the program. Depth-bounded Prolog interpreters, used for theorem-proving, allow efficient testing of hypothesised Horn clause theories. Most importantly, Turing-equivalence of logic programs is allowing a broader range of Machine Learning applications in ILP than was possible with more restrictive representations.

Recent research in ILP has spawned a variety of new theoretical topics.

These include the problem of inverting resolution,^{37,61,54)} inversion of clausal implication,^{22,14,40)} predicate invention,³⁶⁾ closed-world specialisation¹⁾ and U-learnability.⁴²⁾ As with any subject, the diversity of sub-topics can be better understood by following the development of a particular line of ideas. The aims of this paper are firstly to provide a re-appraisal of the development of techniques for inverting deduction, secondly to introduce Mode-Directed Inverse Entailment (MDIE) as a generalisation and enhancement of previous approaches and thirdly to describe an implementation of MDIE in the Prolog* system.

At each stage in the development of ILP there has been an attempt to solve existing technical restrictions of implemented systems. The five main approaches described in this paper are as follows.

- (1) Inverse resolution (IR) in propositional logic,
- (2) IR in first-order definite clause logic,
- (3) determinate relative least general generalisation,
- (4) inverse implication and
- (5) mode-directed inverse entailment.

The paper is structured as follows. First the logical and statistical setting for ILP are introduced (Section 2). This is followed by a synopsis of the results and restrictions for approaches (1) to (4) (Sections 3 to 6). The remainder of the paper (Sections 7 to 12) deals with theoretical and practical aspects of mode-directed inverse entailment. Instructions for obtaining Prolog by anonymous ftp are given in Section 11. The paper closes with a discussion of research issues related to inverse entailment. Standard definitions taken from Logic Programming and ILP are given in Appendix A. In Appendix B a statistical setting for ILP is described. Properties of the subsumption lattice are described in Appendix C. The algorithms used in Prolog are given in Appendix D. A table of Prolog's runtimes various data sets is presented in Appendix E.

§2 Logical and Statistical Setting for ILP

Deductive inference derives consequences E from a prior theory T . Thus if T says that all swans are white, E might state that a particular swan is white. Inductive inference derives a general belief T from specific beliefs E . After observing one or more white swans T might be the conjecture that all swans are white. In both deduction and induction T and E must be consistent and

$$T \models E. \quad (1)$$

The requirement of consistency means that the observation of a black swan rules out conjecture T . Inductive inference is, in a sense, the inverse of deduction. However, deductive inference proceeds by application of sound rules of infer-

* Prolog inverted in the middle.