UNITARY THEORY OF FIELD AND MATTER.


BY MAX BORN.

(From the Department of Physics, Indian Institute of Science, Bangalore.)

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Introduction.

The principal problem of a unitary theory of field and matter is the derivation of the equations of motion of a singularity representing a particle. Several attempts have been made in this direction but none is quite satisfactory. I shall give here a new derivation of the equations of motion of a spinning particle on the basis of the classical treatment of the field equations. This derivation is simple and absolutely rigorous under the suppositions which have to be made so as to give the problem a definite meaning. The chief assumption can be expressed in the usual language of Maxwell's theory in this way: the external field must be constant over the "diameter" of the particle. The unitary field theory does not distinguish between external and internal field; the corresponding supposition is: the (total) field approaches a constant field at a great distance from the singularity. We shall start from a variation principle representing both the motion of the field and of the singularity. Correspondingly it consists of two parts, a space-time integral and a pure time integral. But we shall write the latter also as space-time integral making use of Dirac's $\delta$-function. In this way a great clearness about the physical interpretation of the equations is reached; but the mathematical laws can be easily expressed without symbolic functions and this will be done throughout.

This method leads in the most natural way to the introduction of the spin in the classical theory. Kramers has first shown that a classical spin theory is possible, indeed, and that the quantization of it leads to Dirac's wave equation. The spinning particle is considered by Kramers as a mass point connected with an angular momentum the motion of which is

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described by relativistically invariant formulæ. We show how Kramers' formulæ can be derived from the unitary field theory.

The problem of quantization shall be treated later.

1. Variation principle and field equations.

The Lagrangian \( L(f_{kl}) \) of the field is supposed to be invariant for Lorentz transformation; but we do not assume a special function and rely only on the fact proved in previous publications that there exists such a function for which the energy and momentum of a point charge are finite.

To this Lagrangian of the field we add a Lagrangian of the singularity which we suppose to have the form \( l(\phi_k, f_{kl})\delta \), where \( \delta \) is a symbolic function of the type introduced by Dirac. We assume that in the coordinate system where the singularity is at rest, \( \delta(x, y, z) = 0 \) at every point except in the singularity \( v_o, y_o, z_o \), where \( \delta \) is infinite in such a way that

\[
\int \delta dv = 1, \quad dv = dx dy dz.
\]

The assumption that \( l \) depends explicitly on the potentials \( \phi_k \) does not lead to any difficulties as in the theory of Mie.\(^4\) He introduced the \( \phi_k \) in the Lagrangian of the field: \( L(\phi_k, f_{kl}) \); this leads to contradictions to the fact that the absolute value of the potential in free space has no physical meaning. As the absolute value of the potential in the singularity has a definite meaning, the introduction of the \( \phi_k \) in the Lagrangian of the singularity is permitted.

The variation principle governing field and matter is

\[
(1, 1) \quad \int \{ L(f_{kl}) + l(\phi_k, f_{kl})\delta \} dv dt = \text{Extremum}.
\]

We define the second kind of field components in the usual way by

\[
(1, 2) \quad f_{kl} = \frac{\partial L}{\partial f_{kl}};
\]

further we put

\[
(1, 3) \quad \rho^k = \frac{\partial l}{\partial \phi_k}, \quad m^{kl} = \frac{\partial l}{\partial f_{kl}}.
\]

As the \( f_{kl} \) are connected with the potentials \( \phi_k \) by

\[
(1, 4) \quad f_{kl} = \frac{\partial \phi_k}{\partial x^l} - \frac{\partial \phi_l}{\partial x^k},
\]

one has the identities

\[
(1, 5) \quad \frac{\partial f_{kl}}{\partial x^l} = 0, \quad \text{or} \quad \frac{\partial f_{kl}}{\partial x^m} + \frac{\partial f_{km}}{\partial x^l} + \frac{\partial f_{mk}}{\partial x^l} = 0,
\]

\(^3\) The notations are those used in the previous papers of Born and Infeld, cited above.