INVESTIGATION OF SECONDARY FLOW AND THE RELATED INSTABILITY ON AN INFINITE YAWED CYLINDER WITH SCHUBAUER'S ELLIPSE AS SECTION

BY RAJESHWAR PRASAD SINGH
(Department of Mathematics, Patna University)

AND

K. D. P. SINHA, PH.D.
(Professor of Mathematics, Bihar University, Muzaffarpur)

Received December 14, 1963
(Communicated by Dr. N. S. Nagendra Nath, F.A.S.C.)

ABSTRACT

This paper presents the principal results of a theoretical investigation of the secondary flow and the related instability performed in the laminar incompressible boundary layer on an infinite uniform yawed solid cylinder with Schubauer's ellipse of axial ratio 2.96:1 as the section normal to the leading edge. The secondary flow profiles and the value of the instability criterion are obtained at different points of the wing section and for various angles of sweepback. It is found that in favourable pressure gradients and at pressure minimum, the secondary flow profiles have negative values. In regions of adverse pressure gradients after the pressure minimum the secondary flow changes sign from negative to positive values and have points of inflexion. The change of sign starts from the surface and extends to the edge of the boundary layer downstream. At some points in adverse pressure gradients the secondary flow profiles have double points of inflexion and values of both signs simultaneously. It is found that an adverse pressure gradient produces more powerful secondary flow than a favourable pressure gradient of the same strength.

It is also found that the values of the instability criterion increase with the increasing sweepback whether the pressure gradient is favourable or adverse. At every point of the wing section, there are two values of the criterion for a given sweepback. The effect of adverse pressure gradient on the variation of the criterion is much more pronounced than that of a favourable pressure gradient. It is also seen that the flow is intermittently laminar and turbulent for low values of the chordwise free stream Reynolds number and for low values of sweepback and consists of an irregular sequence of laminar and turbulent regions.
On an infinite yawed cylinder the boundary layer flow is three dimensional.\(^1\) In the case of a three-dimensional boundary layer the external potential flow depends on two co-ordinates in the wall surface and the flow within the boundary layer possesses all three velocity components which, moreover, depend on all three space co-ordinates in the general case. If the stream-lines of the potential motion are straight lines which either converge or diverge then, essentially, the flow differs from a two-dimensional pattern only in that there is a change in the boundary layer thickness. On the other hand, if the potential motion is curved the pressure gradient across the stream-lines of the potential flow impressing itself upon the boundary layer gives rise to additional influences, such as secondary flow: outside the boundary layer the transverse pressure gradient is balanced with the centrifugal force, but within it the centrifugal forces are decreased because of the decreased velocities and consequently, the pressure gradient causes mass to flow inwards, i.e., towards the concave side of the potential stream-lines. At any point in the boundary layer, the velocity component in plane parallel to the span is composed of two components, one parallel to the potential stream-line direction, and the other normal to it. The velocity component parallel to the potential stream-line direction is called the ‘Primary flow’, and the velocity component normal to it is called the ‘Secondary flow’.

Sowerby\(^2\) has shown analytically that in a three-dimensional boundary layer there must be secondary flow unless the stream-lines are straight. He and Loos\(^3\) obtained the solution for the secondary flow in the boundary layer on a plate, where the potential stream-lines are parabolic.

The boundary layer on yawed infinite cylinder and on yawed flat plate was first studied by Sears.\(^4\)

The existence of secondary flow which occurs in the boundary layer of yawed cylinder is important for the aerodynamic properties of swept wings. When yawed or swept-back wings operate at higher lift values the pressure on the suction side near the leading edge shows a considerable gradient towards the receding tip, the effect being due to rearward shift of the aerofoil sections of the wing. The fluid particles which become decelerated in the boundary layer have a tendency to travel in the direction of this gradient, and a secondary flow in the direction of the receding tip results. As demonstrated by measurements performed by R. T. Jones and W. Jacobs\(^1\) (page 198), the boundary layer on the receding portion thickens, the effect leading to premature separation. In aircraft equipped with swept-back wings sepa-