LAMINAR JET OF A COMPRESSIBLE PSEUDO-PLASTIC FLUID

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ABSTRACT

Similarity solutions of the boundary layer equations for compressible pseudo-plastic fluids for plane symmetrical jet are obtained in a closed form. Behaviour of velocity component perpendicular to the axis of the jet is discussed in detail.

INTRODUCTION

Toose (1952) obtained solution in closed form for a plane symmetrical jet of a compressible fluid. Toose's assertion about the behaviour of transverse velocity component was corrected later by Kapur (1958). Kapur (1962, 1963) discussed incompressible two-dimensional jet for pseudo-plastic power law fluids. Here we attempt to extend Kapur's analysis to include compressibility effects. We obtain similarity solutions in a closed form and disuss the behaviour of transverse component of velocity in detail.

BASIC EQUATIONS

We use cartesian co-ordinates taking axis of the jet as x-axis and y-axis perpendicular to it. Origin is some fixed point on jet axis. Suffixes $a$, $j$ and $t$ will denote values in the undisturbed stream, at the orifice and on the axis of the jet respectively.

Two-dimensional boundary layer equations in the absence of pressure gradient for compressible pseudo-plastic power law fluid are easily obtained (Kapur, 1963) as the following:

\begin{align*}
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} &= \frac{\partial}{\partial y} \left[ \mu \left( \frac{\partial u}{\partial y} \right)^{n-1} \frac{\partial u}{\partial y} \right] \quad (1) \\
\rho u \frac{\partial}{\partial x} (c_{pt}) + \rho v \frac{\partial}{\partial y} (c_{pt}) &= \frac{\partial}{\partial y} \left( k \frac{\partial t}{\partial y} \right) + \mu \left( \frac{\partial u}{\partial y} \right)^{n-1} \left( \frac{\partial u}{\partial y} \right)^2 \quad (2) \\
\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) &= 0 \quad (3) \\
\rho t &= \rho_a t_a = \rho_j t_j. \quad (4)
\end{align*}
where \( \rho \) denotes density; \( t \) absolute temperature; \( \mu \) the coefficient of viscosity; \( c_p \) specific heat at constant pressure; \( k \) the coefficient of thermal conductivity; and \((u, v, 0)\) the velocity field. Further we have made use of stress and rate of strain relationship given by

\[
t_{ij} = \mu \left| \sum_{m=1}^{3} \sum_{m=1}^{3} e_{nm} e_{mn} \right|^{(n-1)/2} \epsilon_{ij}
\]

which defines power law fluids, \( n \) being the characteristic of the fluid. \( \mu \) is in general dependent on temperature. This dependence is taken to be the same as by Toose (1952) so that

\[
\frac{\mu}{\mu_j} = \left( \frac{t}{t_j} \right)^n, \quad \frac{1}{2} \leq m \leq 1.
\]

(5)

Since the jet is symmetrical and the transverse component of velocity is to vanish on the jet axis, therefore,

\[
v = \frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0.
\]

(6)

Further axial component of velocity vanishes at an infinite distance from the jet axis, therefore,

\[
u = 0 \quad \text{as} \quad y \rightarrow \infty
\]

(7)

(6) and (7) constitute the boundary conditions of the present problem. We make a change of independent variables from \( x \) and \( y \) to \( x \) and \( \psi \) where \( \psi \) is a stream function defined as

\[
u = \frac{\rho_j}{\rho} \frac{\partial \psi}{\partial y}, \quad \psi = -\frac{\rho_j}{\rho} \frac{\partial \psi}{\partial x}.
\]

It reduces equation (1) to

\[
\rho_j \frac{\partial u}{\partial x} = \frac{\partial}{\partial \psi} \left[ \mu \left| \frac{\rho u}{\rho_j \partial \psi} \right|^{n-1} \frac{\rho u}{\rho_j \partial \psi} \right].
\]

(8)

Further if we denote

\[
U = \frac{u}{u_j}, \quad T = \frac{t}{t_j}, \quad X = \frac{x}{L}, \quad \Psi = \frac{\psi}{\sqrt{u_j v_j} L}
\]

\[
\lambda = \frac{\mu}{\mu_j} = \left( \frac{t}{t_j} \right)^n = T^n, \quad \Omega = \frac{\rho}{\rho_j}.
\]