TEMPERATURE DISTRIBUTION IN A PARALLELOPIPED UNDER THE INFLUENCE OF HEAT SOURCE WITH BOUNDARY CONDITIONS OF SECOND KIND

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ABSTRACT

In the present paper, the general expressions for temperature distribution in an orthotropic plate of finite dimensions have been derived under the influence of an arbitrary volume heat source and arbitrary initial temperature distribution. The boundary conditions prescribed on the faces are of second kind. The solution is obtained by constructing an eigen value problem.

The expression for unsteady temperature distribution consists of two parts—steady and pseudo-steady. The pseudo-steady parts of the solution have been obtained with the help of finite integral transform technique. The general expressions are further studied under different cases.

The whole analysis is presented in the dimensionless form with the help of various dimensionless parameters, viz., $F$, $P$, $K$, etc.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$R_1$</td>
<td>half length of plate in $x_1$ direction.</td>
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<tr>
<td>$R_2$</td>
<td>half length of plate in $y_1$ direction.</td>
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<tr>
<td>$R_3$</td>
<td>half length of plate in $z_1$ direction.</td>
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<td>$R$</td>
<td>characteristic length.</td>
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<td>$F'(x_1, y_1, z_1)$</td>
<td>initial temperature distribution in the plate.</td>
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<tr>
<td>$f_1'(y_1, z_1, t)$, $f_2'(y_1, z_1, t)$</td>
<td>source functions at the surfaces $x_1 = -R_1, x_1 = R_1$ respectively.</td>
</tr>
<tr>
<td>$f_3'(x_1, z_1, t)$, $f_4'(x_1, z_1, t)$</td>
<td>source functions at the surfaces $y_1 = -R_2, y_1 = R_2$ respectively.</td>
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source functions at the surfaces $z_1 = -R_3, z_1 = R_3$ respectively.

$K$ characteristic thermal conductivity.

$K_1, K_2, K_3$ different conductivities in $x_1, y_1, z_1$ directions.

$Q(x_1, y_1, z_1, t)$ internal heat source per unit time and per unit volume.

$Q'(x_1, y_1, z_1, t)$ internal heat source related to $Q$.

$T'(x_1, y_1, z_1, t)$ unsteady temperature distribution defined by equations (1), (2)

$T(x_1, y_1, z_1, t)$ unsteady temperature distribution related to $T'$.

$M(t)$ a function of time.

$(-)$ finite cosine transform of $( )$ defined by (10.1).

$(*$) finite cosine transform of $( )$ defined by (11.1).

$(-)$ finite cosine transform of $( )$ defined by (12.1).

$H(x)$ Heaviside unit function of $x$.

$P_0(x, y, z, F_0)$ Pomerantsev criterion.

$F_0$ Fourier number

$\delta_{ij}$ Kronecker delta

$\rho$ density

$C'$ specific heat capacity.

$x_1, y_1, z_1$ Cartesian coordinates.

INTRODUCTION

In the year 1965 Olcer\textsuperscript{1} presented a general study of unsteady temperature distribution in finite cylinder under the influence of an arbitrary heat source in which he used the boundary conditions of second kind. Further Cobble\textsuperscript{2} studied a transient heat conduction problem in presence of an arbitrary initial temperature distribution by using Hermitian operator. He employed the boundary conditions of convective and homogeneous type.

The purpose of this paper is to present a general study of unsteady temperature distribution in an orthotropic solid in the form of a plate of finite size in presence of heat source under a wide variety of time dependent