A GENERALIZED SQUARE YIELD CONDITION FOR SHELLS OF REVOLUTION

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INTRODUCTION

The plastic analysis of a general rotationally symmetric shell is extremely difficult and relatively little work has been done. The yield surface for a shell whose material obeys the Tresca yield condition was derived by Onat and Prager. The corresponding equations for a shell material obeying the Mises' yield condition were obtained by Hodge. These two yield surfaces are non-linear and the mathematical difficulties are formidable while determining the carrying capacities of structures. Two linear approximations have been proposed by Hodge. In Reference the yield condition for an ideal sandwich shell is used as an approximation for the uniform shell and Reference uses a linear surface which circumscribes the yield surface based on the Tresca condition. Theoretically, it is possible to obtain an exact solution to a rotationally symmetric shell problem by using the above piece-wise linear yield conditions. In practice, the equations become quite complex, and it is doubtful if the resulting labour is worthwhile. In this paper, a much simpler yield surface is proposed. Based on this yield surface, the concentrated collapse load of a simply supported spherical cap is determined.

BASIC EQUATIONS

Figure 1 shows a rotationally symmetric shell element whose state of stress is described by four generalized stresses: The circumferential and meridional bending moments $M_\theta$ and $M_\phi$ and the circumferential and meridional membrane forces $N_\theta$ and $N_\phi$. The shear force $S$ is not considered to be a generalized stress but has the nature of a reaction. The load per unit area of the middle surface of the shell has the components $P_\theta$ in the direction of the meridian and $P_\tau$ in the normal direction. The element has the distance $R_\phi$ from the axis of revolution and its principal radii of curvature are $R_1$ and $R_2$. The generalized stresses must satisfy three equations of equilibrium which may be written.
\[(r_0 n_\phi)' - r_1 n_\theta \cos \phi - r_0 s + r_0 r_1 p_\phi = 0\]
\[r_0 n_\phi + r_1 n_\theta \sin \phi + (r_0 s)' + r_0 r_1 p_r = 0\]
\[k [(r_0 m_\phi)' - r_1 m_\theta \cos \phi] - r_0 r_1 s = 0\]  \(1\)

where we have defined \(n = N/N_0 = N/2\sigma_0 H\), \(m = M/M_0 = M/\sigma_0 H^2\), \(s = S/N_0\), \(p_\phi = LP_\phi/N_0\), \(p_r = LP_r/N_0\), \(r = R/L\); \(\sigma_0\) is the tensile yield stress of the material. The shell is of uniform thickness \(2H\) and radius \(R\) and \(L\) is a typical length. Primes denote differentiation with respect to \(\phi\).

The state of strain is described by four generalized strains which may, in turn, be expressed in terms of the meridional and normal components of the displacement \(V\) and \(W\). The generalized strain rates and the velocities are related by
\[\epsilon_\theta = \frac{1}{r_2} (\dot{V} \cot \phi - \dot{W}), \quad \epsilon_\phi = \frac{1}{r_1} (\dot{V}' - \dot{W})\]
\[x_\theta = -\frac{k}{r_1 r_2} (\dot{V} + \dot{W}'), \quad x_\phi = -\frac{k}{r_1} \left(\frac{\dot{V} + \dot{W}'}{r_1}\right)'.\]  \(2\)

**YIELD CONDITION AND FLOW RULE**

In order to formulate the shell problem, all the constituent equations have to be expressed in terms of the generalized stresses and strain rates.