SELF-RECIPROCAL FUNCTIONS IN THE FORM OF SERIES

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Received June 24, 1958
(Communicated by Dr. T. Venkatarayudu, F.A.SC.)

The object of this note is to find out a few Self-Reciprocal functions in the form of series. Following the usual notation, I denote a function $f(x)$ as $R_\mu$, if it is Self-Reciprocal for Hankel Transforms of order $\mu$ so that it is given by:

$$f(x) = \int_0^\infty J_\mu(xy)f(y) \sqrt{xy} \, dy,$$

where $J_\mu(x)$ is a Bessel function of order $\mu$. If $\mu = -\frac{1}{2}$, $f(x)$ is denoted as $R_c$ while for $\mu = \frac{1}{2}$, $f(x)$ is written as $R_s$.

Dr. Willis has obtained expansions for certain Integrals, in the form of infinite series. Further Dr. Brij Mohan has obtained a class of kernels for Self-Reciprocal functions of Hankel Transforms. On applying these kernels in the Integrals given by Dr. Willis, we find that the expansions in series become Self-Reciprocal functions. Accordingly we show in the following lines that certain Self-Reciprocal functions can be represented as infinite series.

Dr. Willis has shown that

$$\int_0^\infty f(x) e^{-mx} dx = \frac{1}{m} \left\{ f(0) + \frac{f^{(1)}(0)}{m} + \frac{f^{(2)}(0)}{m^2} + \ldots \right\}.$$  

(3.1)

Let $f(x)$ be $R_c$ or $R_s$. Then according to Dr. Brij Mohan the kernel

$$e^x,$$  

(3.2)

transforms

$$R_c(R_s) \text{ into } R_s(R_c).$$

Hence we find that

$$\int_0^\infty e^{mx} f(x) dx,$$

(3.3)

is $R_s(R_c)$ according as $f(x)$ is $R_c(R_s)$.  

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Hence from (3.3) and (3.1), we conclude that
\[ \frac{1}{x} \left\{ f(0) + \frac{f^{(1)}(0)}{x} + \frac{f^{(2)}(0)}{x^2} + \ldots \right\}, \]  
(3.4)
is \( R_s \) (\( R_c \)) according as \( f(x) \) is \( R_c \) (\( R_s \)).

4. Again, Dr. Willis\(^3\) has shown that
\[ \int_0^\infty f(y) \sin xy \, dy = \frac{1}{x} \left\{ f(0) + \frac{f^{(1)}(0)}{x^2} + \frac{f^{(2)}(0)}{x^4} + \ldots \right\}. \]  
(4.1)
Let \( f(x) \) be \( R_s \). Then by the definition of an \( R_s \) function we find that
\[ \int_0^\infty f(y) \sin xy \, dy = f(x). \]  
(4.2)
Hence from (4.2) and (4.1), we conclude that
\[ \frac{1}{x} \left\{ f(0) + \frac{f^{(1)}(0)}{x^2} + \frac{f^{(2)}(0)}{x^4} + \ldots \right\}, \]  
(4.3)
is \( R_s \) if \( f(x) \) is \( R_s \).

5. Further, it has been shown by Dr. Willis\(^3\) that
\[ \int_0^\infty f(y) J_0(xy) \, dy = \frac{1}{x} \left\{ f(0) - \frac{f^{(1)}(0)}{2} \frac{1}{1 \cdot x^2} + \frac{1}{2^2} \frac{1}{2 \cdot x^4} \right. \]
\[ \left. \quad - \frac{1}{2^3} \frac{3}{3 \cdot x^6} + \ldots \right\}, \]  
(5.1)
Let \( f(x) \) be \( R_c \) or \( R_s \).

Dr. Brij Mohan\(^1\) has shown that the kernel
\[ J_0(x), \]  
(5.2)
transforms
\( R_c \) (\( R_s \)) into \( R_s \) (\( R_c \)).
Hence from (5.1) and (5.2), we conclude that
\[ \frac{1}{x} \left\{ f(0) - \frac{f^{(1)}(0)}{2} \frac{1}{1 \cdot x^2} + \frac{1}{2^2} \frac{1}{2 \cdot x^4} \right. \]
\[ \left. \quad - \frac{1}{2^3} \frac{3}{3 \cdot x^6} + \ldots \right\}, \]  
(5.3)
is \( R_s \) (\( R_c \)) according as \( f(x) \) is \( R_c \) (\( R_s \)).