A NOTE ON CASCADE THEORY WITH IONISATION LOSS

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Received January 3, 1957

ABSTRACT

The cascade theory of cosmic ray showers including ionisation loss is dealt with on the basis of the new approach suggested by us in an earlier contribution and an explicit Mellin transform solution is obtained for the mean number of particles produced in an infinite thickness of matter.

Some years ago, Bhabha and Chakrabarthy (1943) obtained and discussed the equations of the cascade theory expressing the mean behaviour of the particles when collision loss is also taken into account. If \( f(E; t) \) and \( g(E; t) \) are the mean numbers in the energy interval \( dE \) of electrons and photons respectively and \( R(E'|E) \) and \( \rho(E'|E) \) are the cross-sections for radiation and pair-production respectively, the equations are

\[
\frac{df(E; t)}{dt} = -f(E; t) \int_0^E R(E'|E) dE' + \int_E^\infty f(E'; t) R(E'|E) dE' + 2 \int_0^E g(E'; t) \rho(E'|E) dE' + \frac{\beta \cdot df(E; t)}{dE}
\]

\[
\frac{dg(E; t)}{dt} = -g(E; t) \int_0^E \rho(E'|E) dE' + \int_E^\infty f(E'; t) R(E' - E|E') dE'
\]

with the initial conditions

\( f(E; 0) = \delta(E - E_0) \)

where \( \delta \) is the Dirac delta function and

\( g(E; 0) = 0 \)

corresponding to a shower excited by a single electron of energy \( E_0 \). In conformity with the theory of 'product densities' (which is usually useful for obtaining higher moments of the distribution) we recognise \( f \) and \( g \) as

* For a formulation of the theory of product densities, see Ramakrishnan (1950).
the product densities of degree one of electrons and photons at $t$. The collision loss in traversing unit thickness of matter is assumed to be a constant, $\beta$. Defining the Mellin transforms of $f(E; t)$ and $g(E; t)$ as

$$p(s; t) = \int_0^\infty f(E; t) E^{s-1} dE$$

$$q(s; t) = \int_0^\infty g(E; t) E^{s-1} dE$$

according to Bhabha and Chakrabarthy, equations (1) and (2) can be reduced to

$$\frac{dp(s; t)}{dt} = -A_s p(s; t) + B_s q(s; t) - (s - 1) \beta p(s - 1; t)$$

$$\frac{dq(s; t)}{dt} = C_s p(s; t) - D q(s; t)$$

$A_s$, $B_s$, $C_s$ and $D$ are given by (see Bhabha and Chakrabarthy, 1943)

$$A_s = \left(\frac{4}{3} + a\right) \left\{ \frac{d}{ds} \log \left[ s + \gamma - 1 + \frac{1}{s} \right] + \frac{3}{2} - \frac{1}{s (s + 1)} \right\}$$

$$B_s = 2 \left\{ \frac{1}{s} - \left(\frac{4}{3} + a\right) \frac{1}{(s + 1) (s + 2)} \right\}$$

$$C_s = \frac{1}{s + 1} + \left(\frac{4}{3} + a\right) \frac{1}{s (s - 1)}$$

$$D = \frac{7}{9} - \frac{1}{6} a$$

We note that (7) is a difference-differential equation where the difference relates to the complex variable $s$ and hence cannot be solved by direct iteration. Moreover the dependence on $t$ makes the solution much more difficult. A simple question that can be asked is: What is the spectrum at $t = \infty$? Unfortunately the product density functions $f(E; t)$ and $g(E; t)$ tend to zero as $t \to \infty$ (for $E > 0$) and thus nothing interesting can be obtained. However on the basis of the new approach suggested by us in a previous paper (1956) we can ask for the mean number of particles produced in the entire shower, each of the particles having an energy greater than $E_c$ at the point of its production. We shall show that explicit Mellin transform solution for the mean number of particles in this case can indeed be obtained using standard mathematical techniques.