Equalization strategies for transmission over space and time

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Dedicated to Univ.-Prof. i. R. Dipl.-Ing. Dr. techn. Johann Weinrichter on occasion of his 65th birthday

An overview of equalization schemes applicable to point-to-point communication over MIMO ISI channels is given, i.e., channels which suffer from both multiuser interference between the data streams transmitted in parallel and intersymbol interference due to the dispersive (frequency-selective) nature of the channel. Spatial, temporal, and combined spatial/temporal equalization strategies for dealing with these types of interferences are discussed. In particular, linear and decision-feedback equalization, and equalization based on singular value/eigenvector decomposition and lattice basis reduction, respectively, are treated. The underlying mathematical principle, utilized in these schemes, is stated in each case. Via numerical simulations, the performance of selected equalization strategies is compared.

Keywords: MIMO channel; equalization; intersymbol interference; multiuser interference; point-to-point communication

Entzerrungsstrategien für die Übertragung über Raum und Zeit.


Schlüsselwörter: MIMO-Kanal; Entzerrung; Intersymbolinterferenzen; Mehrbenutzerinterferenzen; Punkt-zu-Punkt-Übertragung

1. Introduction

It is now widely accepted that for achieving very high spectral efficiencies – and in turn data rates – the use of antenna arrays in transmitter and receiver, hence resorting to multiple-input/multiple-output (MIMO) communication, is the most promising strategy.

Over the last years, mainly flat-fading MIMO channels have been studied. But when increasing data rates even more by using larger and larger bandwidths, the frequency selectivity of the channels becomes more and more pronounced. Then, in addition to multiuser interference (MUI) between the data streams transmitted in parallel over the antennas, intersymbol interference (ISI) due to channel dispersion occurs. Hence, next-generation high-rate transmission schemes have to deal with such MIMO ISI channels.

The aim of the present article is to give an overview of equalization schemes applicable to communication over MIMO ISI channels. For that, we first treat both phenomena, MUI, i.e., spatial interference, and ISI on single-input/single-output (SISO) channels, i.e., temporal interference, separately. Spatial and temporal equalization strategies for dealing with these types of interferences are discussed. In particular, linear equalization, decision-feedback equalization, equalization based on singular value/eigenvector decomposition, and lattice basis reduction are treated, and the respective mathematical principle, on which the schemes are based, is stated. Then, for each strategy, the combination to combined spatial/temporal equalization is presented.

Please note, throughout the article we restrict ourself to point-to-point communication, i.e., transmitter and receiver, respectively, have full access to all antenna elements ( Bölcskei, Weinrichter, 2002). This is in contrast to multipoint-to-point (typically the unlink) or point-to-multipoint (typically the downlink) scenarios. However, most of the presented equalization strategies are applicable in either of the mentioned situations, dependent on whether joint transmitter or joint receiver side processing is required. Moreover, maximum data rate is desired. Hence, no methods for increasing diversity at the price of data rate – so-called space-time codes, e.g. (Badic et al., 2004 – are considered.

Sect. II gives the general MIMO ISI channel model. In Sect. III, the equalization strategies are explained and compared. Results from numerical simulations are given in Sect. IV, and Sect. V concludes the paper.
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2. Channel model

Point-to-point transmission using antenna array with \( N_t \) transmit and \( N_r \) receive antennas is considered. Over the antennas, mutually independent\(^1\) i.i.d. sequences \( \{ a_k \} \) of \( T \)-spaced data symbols \( a_k \), \( k \in \mathbb{Z}, \mu_1 = 1, 2, \ldots, N_t \) taken from (possibly different) signal constellations \( \mathcal{A}_k \) (variance normalized to \( \sigma_k^2 \) \( \mathbb{E} \{ |a_k|^2 \} = 1 \)). The data symbols are combined into the vector \( \mathbf{a} \) \( \in \mathbb{C}^{N_t} \) \( [a_1, \ldots, a_{N_t}]^\top \). The actual transmission channel consists of three parts. First, at the transmitter, pulse shaping is performed to obtain a continuous-time transmit signal. Then the signal is modulated to the radio frequency band. Second, the radio frequency signal is transmitted over the antenna array and propagates through the channel. In high-rate systems using large bandwidths, the frequency selectivity of the channel becomes important. Hence, the individual channel from each transmit to each receive antenna have to be characterized by (continuous-time) impulse responses rather than single (flat-fading) scaling gains. At the receive antennas, the superpositions of all linearly distorted transmit signals and the unavoidable additional noise are present. Third, the radio frequency receive signals are demodulated and passed through a bank of receive filters, at the output of which \( T \)-spaced sampling is performed. Note, in (Van Etten, 1976) it has been proved that the optimum (with respect to a wide range of criteria) receive front-end consists of a bank of matched filters for the cascade of pulse shaping filters and continuous-time channel.

All further processing steps may then be performed by a discrete-time filter, succeeding the \( T \)-spaced sampling.

Combining the three above mentioned parts, an end-to-end \( T \)-spaced discrete-time channel model (given in the equivalent complex baseband domain (Van Trees, 1971; Proakis, 2001)) between vector of data symbols \( \mathbf{a} \) \( \in \mathbb{C}^{N_t} \) and vector of received symbols \( \mathbf{y} \) \( \in \mathbb{C}^{N_r} \) \( \{ y_1, \ldots, y_{N_r} \} \) can be established. Thereby we assume that the receiver (in particular the discrete-time filter part) is adjusted such that the multiplied whitened matched filter (Van Etten, 1976) is implemented. This receive filter \( \mathbf{h} \) provides a lossless transition from the continuous-time to the discrete-time model (even though sampling theorem is usually not met), and if \( \mathbf{h} \) is the optimal starting point for a wide class of subsequent equalization strategies (Fischer, Huber, Windpassinger, 2003). Assuming additionally that the channel is (approximately) constant over each transmission burst, each individual channel from each transmit to each receive antenna\(^2\) is characterized by a time-invariant, causal impulse response \( h_k \), \( k \geq 0 \). Due to the multiplied whitened matched filter, the additive discrete-time Gaussian noise is spatially and temporally white with variance \( \sigma_n^2 = N_r \bar{x}_k \), i.e., the correlation matrix of the additive noise vector \( \mathbf{n} \) \( \in \mathbb{C}^{N_r} \) \( \{ n_1, \ldots, n_{N_r} \} \) reads \( \mathbf{\Phi}_n = \text{diag} \{ \mathbb{E} \{ n_1^2 \}, \ldots, \mathbb{E} \{ n_{N_r}^2 \} \} \). The taps of the individual impulse responses are advantageously combined into \( N_r \times N_t \) tap matrices

\[ \mathbf{H}_k \triangleq \begin{bmatrix} h_{11} \ [k] & \cdots & h_{1N_t} \ [k] \\ \vdots & \ddots & \vdots \\ h_{N_r1} \ [k] & \cdots & h_{N_rN_t} \ [k] \end{bmatrix}. \] (1)

This channel is completely characterized by the matrix channel transfer function

\[ \mathbf{H} (z) = \mathbf{H} \odot \mathbf{z} \quad \text{or} \quad \mathbf{y} = \mathbf{H} \mathbf{a} + \mathbf{n}, \tag{2} \]

where \( L \) is the maximum length of the impulse responses \( \mathbf{H} (z) \) is a matrix polynomial of degree \( L - 1 \). Thus, the action of the discrete-time linear dispersive vector channel is described by

\[ \mathbf{y} \ [k] = \mathbf{\sum}_{\ell=-\infty}^{\infty} \mathbf{H}_\ell \mathbf{a} \ [k - \ell] + \mathbf{n} \ [k]. \tag{3} \]

Figure 1 visualizes the channel impulse response, which is dependent on the three parameters transmit antenna \( \mu_1 \), receive antenna \( \nu \), and discrete time index \( k \). Transmitting a single unit impulse over antenna \( \mu \), the impulse response for the given \( \nu = 1, \ldots, N_r \) and \( k \geq 0 \) is observed which, in the visualization, lies in the \( \nu \times k \) plane.

The above mentioned channel model contains two special cases: First, if all individual channel impulse responses are of length 1, only \( H_{11} \) has non-zero entries and the common flat-fading MIMO channel model results. Second, for \( N_t = N_r = 1 \) a SISO ISI channel is present.\(^3\) Both extreme cases are visualized in Fig. 1, too. In the first case, only spatial interference is present; the data symbols transmitted in parallel at the same time interfere (also called (multi)user interference). For reliable detection of the transmitted symbols, equalization with respect to the spatial dimension has to be performed. In the second scenario, temporal interference (intersymbol interference) is active; data symbols of a single user transmitted in sequence interfere. Here, equalization with respect to time is required. In the general setting, where both spatial and temporal interferences occur, joint spatial/temporal equalization has to be implemented.

3. Characterization of equalization strategies

In this section, equalization strategies for spatial and temporal equalization are discussed. We thereby first concentrate on pure spatial and pure temporal equalization. Then, combined spatial/temporal equalization strategies are presented. In each case, perfect channel knowledge at the receiver (if specified at the transmitter, too) is assumed.

Please note, in this article we restrict ourselves to symbol-by-symbol detection schemes. We hence do not consider maximum-likelihood detection (MLD, spatial) (Proakis, 2001), maximum-likelihood sequence estimation (MLSE, temporal) (Forney, 1972), or joint space/time MLSE, which is optimum for MIMO ISI channels (Van Etten, 1976). Such schemes are in general much too complex. Assuming a MIMO ISI channel with parameters \( N_t, N_r, \) and \( L \), and cardinalities \( M \triangleq |\mathcal{A}| \) of the signal constellations, a Viterbi-algorithm-based MLSE scheme would have \( L^M \) \( M^{L-1} \) states and \( M^L \) branches leaving and entering each state. Even for moderate values (e.g., \( N_t = 3, L = 5, \) and \( M = 16 \)), the number of states \( (16^2 \times 2.8 \times 10^8) \) and branches \( (16^6 \times 4096) \) i.e., \( 16^6 \times 16^6 \times 3 \) trellis branches would have to be visited per decoded bit of information) become prohibitively large. We hence concentrate on equalization for

\(^1\) In case of transmitter side preprocessing, the actual transmit symbols, denoted as \( x_k \), are calculated from the mutually independent i.i.d. data symbols \( a_k \), and hence may exhibit correlations.

\(^2\) Notation: Matrices are printed in boldface uppercase letters, column vectors in boldface lowercase letters, and scalars in regular lowercase letters. \( \mathbf{^t} \) transposition; \( \mathbf{^H} \) Hermitean (i.e., conjugate transpose); \( A^{-1} \) inverse of the Hermitean transpose of a square matrix \( A \); \( I \) identity matrix; \( x \) abbreviates \( (x)^{\mathbb{C}} \); \( \mathbb{E} \{ \cdot \} \): expectation; \( \mathcal{F} \! \{ \cdot \} \) Fourier transform with respect to \( \nu \).

\(^3\) The terms transmit antenna and receive antenna are used for the discrete-time signals, too, as it is usual in literature. However, please keep in mind that in practice only the continuous-time radio-frequency signals are actually present at the antennas.