HYDROMAGNETIC RAYLEIGH PROBLEM IN A ROTATING FLUID

A. RAPTIS
University of Ioannina, Department of Mechanics
Ioannina, Greece

and

A. K. SINGH
Banaras Hindu University, Department of Mathematics
Varanasi-221005, India

(Received 18 October 1984)

An analysis of the MHD Rayleigh problem of a rotating fluid above a plate is carried out, when the magnetic Reynolds number of the flow is small and the magnetic field is fixed on the plate. An exact solution is obtained with the help of the Laplace transform technique. It is found that the effect of the magnetic field increases the primary flow and decreases the secondary flow.

1. Introduction


The object of the present paper is to study the hydromagnetic Rayleigh problem, when the magnetic field is fixed on the moving plate and the magnetic Reynolds number of the flow is small. The solution of the problem is obtained with the help of the Laplace transform technique.

2. Mathematical analysis

The equation of continuity and the equation of motion along with the Maxwell’s equations and Ohm’s law for unsteady flow of viscous, incompressible and electrically conducting fluid are:

\[ V \cdot V = 0, \]  

(1)
\[ \frac{\partial \mathbf{V}}{\partial t'} + (\mathbf{V} \cdot \nabla) \mathbf{V} + 2\Omega \hat{z}_0 \times \mathbf{V} = -\frac{1}{\rho} \mathbf{V} P + \nu \nabla^2 \mathbf{V} + \frac{1}{\rho} \mathbf{J} \times \mathbf{B}, \]  
\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J}, \]  
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t'}, \]  
\[ \nabla \cdot \mathbf{B} = 0, \]  
\[ \mathbf{J} = \sigma (\mathbf{V} \times \mathbf{B} + \mathbf{E}), \]

where \( \rho, \nu, \sigma, \mu_0, \mathbf{V}, \mathbf{J}, \mathbf{B}, \mathbf{E}, P, \hat{z}_0 \) and \( \Omega \) are respectively the density, the kinematic viscosity, the fluid conductivity, the magnetic permeability, the velocity vector, the current density, the magnetic field, the electric field, the pressure including the centrifugal term, the unit vector along the \( z' \)-axis and the constant angular velocity of the fluid about the \( z' \)-axis.

We consider the unsteady flow generated in a semi-infinitely extended fluid bounded by an infinite flat plate at \( z' = 0 \), subjected to a uniform magnetic field density \( B_0 \) which is perpendicular to the plate. Both the fluid and the plate are in a state of solid body rotation with the constant angular velocity \( \Omega \) about \( z' \)-axis, which is perpendicular to the plate, and at time \( t' = 0 \) the plate is started impulsively with a uniform velocity \( U_0 \) (in the rotating frame of reference) at the direction of the \( x' \)-axis and the velocity field is of the form \( \mathbf{V}(u, u, 0) \).

Since the plate is infinite in length all physical quantities will be functions of \( z' \) and \( t' \). Then the equation of the conservation of electric charge \( \nabla \cdot \mathbf{J} = 0 \) gives \( J_z = \text{constant} \), where \( \mathbf{J} \equiv (J_x, J_y, J_z) \). This constant is equal to zero \( (J_z = 0) \) when the plate is electrically non-conducting. Since the magnetic Reynolds number of the flow is small, the induced magnetic field can be neglected in comparison to the applied magnetic field. Hence we can take \( \mathbf{B} \equiv (0, 0, B_0) \). Also we assume that the electric field is equal to zero.

Since the magnetic field is fixed on the moving plate it will be \[ J_x = \sigma B_0 \hat{z}', \quad J_y = \sigma B_0 (u' - U_0). \]

By taking into account these assumptions Eq. (2) gives:

\[ \frac{\partial u'}{\partial t'} - 2\Omega u' = \nu \frac{\partial^2 u'}{\partial z'^2} - \frac{\sigma B_0^2}{\rho} (u' - U_0), \]  
\[ \frac{\partial u'}{\partial t'} + 2\Omega u' = \nu \frac{\partial^2 u'}{\partial z'^2} - \frac{\sigma B_0^2}{\rho} u'. \]

The initial and boundary conditions are:

\[ \text{For } t' \leq 0: \quad u' = 0, \quad \nu' = 0 \quad \text{for all } z'. \]