IMPURITY INDUCED $T_C = 0^\circ K$ SUPERCONDUCTIVITY

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Our earlier result, representing the dependence of critical temperature of superconducting transition on the concentration of dia- and paramagnetic impurities is applied to the case when $T_C$ approaches absolute zero owing to impurities. We calculated the rate of spin flip scatterings $\Delta \Gamma_{so}$ for an electron at the Fermi surface, when either the scattering processes or the change in the Fermi surface topology have a dominant role. The expressions obtained were verified by a comparison with experimental data and by numerical estimation.

Introduction

In a previous paper [1] we examined the effect of para- and diamagnetic impurities on the temperature of superconducting transition. Assuming a Lifshitz singularity in the density of normal single-electronic states, we obtained the next equation for the determination of the critical temperature $T_C$

$$\lambda \nu_0(\epsilon_F^0) \Re \int_0^{\omega_p} d\omega \frac{\omega}{\omega + i\Gamma_s} \frac{\lambda M}{\Re \int_0^{\omega_p} d\omega \frac{\omega}{\omega + i\Gamma_s}} \left(\frac{\alpha - \omega}{\alpha - \omega} + \frac{i}{\alpha - \omega}\right) = 1,$$

where the first term is the result of the ABRIKOSOV—GORKOV theory [2] and the second term is due to the singularity. In this formula the symbols mean: $\lambda$ the potential of electron—electron interaction, $\nu_0(\epsilon_F^0)$ the density of the regular normal single-electronic states on the Fermi surface of the pure metal $\epsilon_F^0$, $\omega_D$ the Debye frequency, $\Gamma_n$ and $\Gamma_s$ are the rates of normal and spin flip scatterings for an electron at the Fermi surface (proportional to the impurity concentration), $M$ is a constant depending on the effective mass of electron:

$$M = \frac{m_1 \sqrt{m_2}}{2\pi^2}$$

and

$$\alpha = \gamma_d \Delta Z_d \Gamma_{nd} + \gamma_p \Delta Z_p \Gamma_{np} - i \frac{\Gamma_n + \Gamma_s}{2} + \epsilon_F^0 - \epsilon_c,$$

$$\alpha^* = \gamma_d \Delta Z_d \Gamma_{nd} + \gamma_p \Delta Z_p \Gamma_{np} + i \frac{\Gamma_n + \Gamma_s}{2} + \epsilon_F^0 - \epsilon_c,$$
where \( m_1 \) and \( m_3 \) are the diagonal elements of the effective mass tensor, \( \Delta Z_d \) and \( \Delta Z_p \) are the differences in the valence of dia- and paramagnetic impurities and the normal metal, \( \gamma_d \) and \( \gamma_p \) are constants, \( \varepsilon_c \) is the critical value of electron energy.

**Superconductivity at \( T_c = 0 \, ^\circ K \)**

The value of \( T_c \) may be decreased to absolute zero owing to the effects of impurities. This is due to paramagnetic impurities. In this case, in Eq. (1) we have

\[
\text{th} \frac{\omega}{2T_c} = 1.
\]

If \( \Gamma_s = \Gamma_{s0} \), where \( \Gamma_{s0} \) is the value at which \( T_c = 0 \) according to Abrikosov—Gorkov theory, then

\[
\lambda v_0(\varepsilon_F) \frac{\omega}{\omega + i\Gamma_{s0}} = 1.
\]

Writing \( \Gamma_s = \Gamma_{s0} + \Delta \Gamma_{s0} \) and using the iteration method we have, in an approximation of first order,

\[
\Delta \Gamma_{s0} = C \text{Re} \left\{ -\int_0^{\omega_p} d\omega \frac{i(\Gamma_{n0} + \Gamma_{s0}) + 2\omega}{(\omega + i\Gamma_{s0})[\sqrt{\omega_0 - \omega} + \sqrt{\omega_0 + \omega}]} \right\},
\]

where

\[
C = -\frac{M}{v_0(\varepsilon_F)} \frac{\Gamma_{s0}(\omega_D^2 + \Gamma_{s0}^2)}{\omega_D^2}.
\]

Taking into account that \( \Gamma_{n0}/\omega_D \sim 1 \), the absolute values of square roots in the denominator of Eq. (5) are large, we may write, approximately,

\[
\Delta \Gamma_{s0} = C \text{Re} \left\{ -\frac{i(\Gamma_{n0} + \Gamma_{s0}) \ln(\omega + i\Gamma_{s0}) - 2\omega + 2i\Gamma_{s0}(\omega + i\Gamma_{s0})}{\sqrt{\omega_0 - \omega} + \sqrt{\omega_0 + \omega}} \right\}.
\]

The value of \( \Delta \Gamma_{s0} \) we calculate in two important limiting cases of experimental interest.

a) If the scattering processes dominate, e.g. \( \gamma_p \ll 1 \), and assuming, that \( \Gamma_{nd} = 0 \), then expression (6) has the form

\[
\Delta \Gamma_{s0} = C \text{Re} \left\{ \frac{i(\Gamma_{s0} - \Gamma_n) \ln(\omega_D + i\Gamma_{s0}) - 2\omega_D}{(1 + i) \sqrt{\omega_D + \frac{1}{2} i(\Gamma_n + \Gamma_{s0})}} - \frac{i(\Gamma_{s0} - \Gamma_n) \ln i\Gamma_{s0}}{\sqrt{2} i \sqrt{\frac{1}{2} (\Gamma_n + \Gamma_{s0})}} \right\}.
\]