The governing principle of dissipative processes is formulated for unsteady heat conduction phenomenon. The dual field method is applied to find the temperature distribution in a finite insulated rod whose ends are maintained at a constant temperature. The variational solution obtained by this new method is in excellent agreement with the exact solution given by Carslaw and Jaeger. The result is also obtained in the force representation of the governing principle which is exactly similar to that obtained by local potential method.

Introduction

On the basis of non-equilibrium theory of irreversible processes Gyarmati [1, 2] formulated a variational principle which describes the evolution of dissipative processes in time and space. The formulation which is called the governing principle of dissipative processes is written in most general form as

$$\delta \int_V [\sigma - \mathcal{Y} - \Phi] \, dv = 0. \quad (1)$$

Here $\sigma$ denotes the entropy production inside the system and it is a bilinear function of thermodynamic forces $\Delta \Gamma_i$ and current $J_i$ i.e.

$$\sigma = \sum_{i=1}^{f} J_i \cdot \nabla \Gamma_i \geq 0. \quad (2)$$

$\Gamma_i$ are the state parameters, the gradients of which are the thermodynamic forces. In the linear Onsager theory, the currents are linear functions of the forces, i.e.

$$J_i = \sum_{k=1}^{f} L_{ik} \, \nabla \Gamma_k, \quad \nabla \Gamma_i = \sum_{k=1}^{f} R_{ik} \, J_k, \quad (i = 1, 2, \ldots f) \quad (3)$$

where the constant coefficients $L_{ik}$ and $R_{ik}$ are the conductivities and resistances, respectively, and these satisfy the famous reciprocal relations

$$L_{ik} = L_{ki}, \quad R_{ik} = R_{ki} \, (i, k = 1, 2, \ldots f). \quad (4)$$
Ψ and Φ are the local dissipation potentials which are defined as [1, 2]

\[ \Psi(\nabla \Gamma, \nabla \Gamma) = \frac{1}{2} \sum_{i,k=1}^{f} L_{ik} \nabla \Gamma_i \cdot \nabla \Gamma_k \geq 0 , \quad (5) \]

\[ \Phi(J, J) = \frac{1}{2} \sum_{i,k=1}^{f} R_{ik} J_i \cdot J_k \geq 0 . \quad (6) \]

These functions, in Onsager's linear theory, are equal to half of the entropy production for real processes, i.e. Ψ and Φ are the local measures of irreversibility. Using (2), (5) and (6), the variational principle (1) can be given in the following detailed form

\[ \delta \int_v \left[ \sum_{i=1}^{f} J_i \cdot \nabla \Gamma_i - \frac{1}{2} \sum_{i,k=1}^{f} L_{ik} \nabla \Gamma_i \cdot \nabla \Gamma_k - \frac{1}{2} \sum_{i,k=1}^{f} R_{ik} J_i \cdot J_k \right] dv = 0 . \quad (7) \]

It should be noted that the principle (7) is operative if and only if the balance equations

\[ \dot{\varrho}_i + \nabla \cdot J_i = \sigma_i \quad (i = 1, 2, \ldots f), \quad (8) \]

are regarded as auxiliary conditions for whose variations the restrictions

\[ \delta (\dot{\varrho}_i - \sigma_i) = -\delta \Delta \cdot J_i = -\nabla \cdot J \delta \dot{i} \quad (i = 1, 2, \ldots f) \quad (9) \]

are valid. Here \( \dot{\varrho}_i \) is the partial time derivative of the density \( \varrho_i \) and \( \sigma_i \) is the rate of production of the transport quantities.

The principle is already used extensively for the derivation of equations of heat conduction, diffusion etc. Recently Singh [3, 4, 5] applied this principle to get the variational solution of the Bénard convection. The critical wave and Rayleigh numbers for the linearised Bénard convection were obtained when the principle of exchange of stability is valid. In the following, the principle is applied to get the solution of the time dependent process of heat conduction in a finite insulated rod, the ends of which are maintained at constant temperature say zero. Assuming that the initial temperature is given by \( T_0(1-x^2) \) where \( T_0 \) is constant and \( x \) is the distance measured along the rod, the temperature distribution which depends on both time and position is obtained. The result obtained using the universal form of principle is quite close to the exact result given by Carslaw and Jaeger [6]. The result was also obtained in force representation which is same as obtained by Schechter [7].

Acta Physica Academiae Scientiarum Hungaricae 43, 1977