RELATIVISTIC CORRECTIONS TO CHARGE TRANSFER SCATTERING

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Relativistic corrections to charge transfer scattering of proton on hydrogen has been calculated in a semi-relativistic treatment and a qualitative discussion of the result has been given.

1. Introduction

In recent years several attempts (Carpenter et al., Chen and Hambro [1], Das [2], Dettmann et al. [3], Drisko [4], McCarroll et al. [5], Chen [6] have been made to find a correct form for the charge transfer scattering of proton on hydrogen. One interesting result in this connection is the second Born Calculation of Drisko [4]. The works of Carpenter and Tuan [7] and of Chen, Chen and Kramers [8] have raised again a question as to the energy region where the result of Drisko is valid. From two recent articles of one of the authors (Das [2, 9]) one may form some idea about this energy region. In this article we wish to see up to what energy relativistic effects may be neglected and results of Drisko may be applied and secondly to find in what way the formula of Drisko will be affected at a very high energy. In this discussion for simplicity, we take non-relativistic expressions for wave functions with the appropriate relations between energy and momentum.

2. Calculation

The expression for the charge transfer scattering process

\[ H^+ + H (1s) \rightarrow H (1s) + H^+ \]  

may be written (Das [10]) to second Born approximation in the form

\[ T^{(2)} = T_{BK} + T_{TR} + T_{B'}. \]
where $T_{BK}$ and $T_{B2}$ are same as in the article of Das [9] and the term $T_{Tr}$ is given by (in the notations of Das [10])

\[
\sum_{i_1} \Phi_f | H_{Tr} | \Phi_{i_1} > \frac{1}{E_i - E_{i_1} + i\delta} \Phi_{i_1} | H_{Tr} | \Phi_i > \tag{3a}
\]

\[
+ \sum_{i_2} < \Phi_f | H_{Tr} | \Phi_{i_2} > \frac{1}{E_i - E_{i_2} + i\delta} < \Phi_{i_2} | H_{Tr} | \Phi_i > , \tag{3b}
\]

where

\[
H_{Tr} = -\frac{e}{mc} \left\{ \frac{\hbar}{i} \cdot \mathbf{\partial} \right\} \overrightarrow{A}(\overrightarrow{R}_1) - \frac{e}{Mc} \left\{ \frac{\hbar}{i} \cdot \mathbf{\partial} \right\} \overrightarrow{A}(\overrightarrow{R}_2)
\]

\[
+ \frac{e}{mc} \sum \int dx_e \psi_e^* (x_e) \left[ \frac{\hbar}{i} \cdot \mathbf{\partial} \right] \psi_e (x_e) \overrightarrow{A}(\overrightarrow{r}_e)
\]

\[+ \text{second order terms in } e , \]

and $\Phi_{i_1}$, $\Phi_{i_2}$ are possible intermediate states. (Wave functions are approximated by non-relativistic expressions.)

One photon intermediate states contribute to the expressions (3a) and (3b). We choose as new independent variables $\overrightarrow{R}_e - \overrightarrow{r}_e$ and $\overrightarrow{r}_e$ where

\[
\overrightarrow{R} = \frac{M\overrightarrow{R}_1 + M\overrightarrow{R}_2 + m\overrightarrow{r}}{2M + m},
\]

\[
\overrightarrow{r}_{1e} = \overrightarrow{R}_1 - \overrightarrow{r}_e ,
\]

\[
\overrightarrow{r}_{2e} = \overrightarrow{R}_2 - \overrightarrow{r}_e
\]

are the position vectors of the centre of mass and of the protons from that of the electron.

The dominant contribution to expression (3a) is

\[
T_{Tr}^{(1)} = \sum \frac{1}{E_i - E_{i_1} + i\delta} \times T_{f_{i_1} i_1} \times T_{f_{i_1} i} , \tag{5}
\]

where

\[
T_{f_{i_1}} = \frac{e}{m} \left( \frac{2\pi\hbar}{c\alpha} \right)^{1/2} \int \Phi^* f(x) \left( \frac{M}{2M + m} \overrightarrow{r}_{1e} - \frac{M}{2M + m} \overrightarrow{r}_{1e} \right) \times \left[ -\varepsilon \left( \frac{\hbar}{i} \cdot \overrightarrow{r}_{1e} \right) - \varepsilon \left( \frac{\hbar}{i} \cdot \overrightarrow{r}_{2e} \right) \right] \Phi_{i_1} \tag{6a}
\]

and

\[
T_{i_1 i} = -\frac{e}{M} \left( \frac{2\pi\hbar}{c\alpha} \right)^{1/2} \int \Phi^* f(x) \left( \frac{\hbar}{i} \cdot \overrightarrow{r}_{1e} \right) \times \left[ \varepsilon \left( \frac{M + m}{2M + m} \overrightarrow{r}_{1e} - \frac{M + m}{2M + m} \overrightarrow{r}_{1e} \right) \right] \Phi_i . \tag{6b}
\]