A differential geometric analysis has been made with respect to free surfaces in both undeformed and deformed crystals. It has been shown that such well-defined tensor quantities as distortion, torsion, anholonomic object, Burgers vector and Burgers circuit may be defined with respect to such a surface.

Introduction

It has already been shown that internal surfaces such as grain boundaries [1] and two-phase interfaces [2] can be described in terms of well-defined dislocation arrays. A problem closely related to this concerns the nature of a free surface. In particular, one wishes to know the exact configuration of the Burgers circuit and the corresponding tensor quantities associated with these free surfaces. The purpose of the present study is thus to employ the techniques of differential geometry in the analysis of this particular problem.

Burgers circuit associated with a simple surface

Consider the perfect crystal shown in Fig. 1a. A reference circuit may now be constructed within such a crystal as shown by the arrows along the path 1-2-3-4-5-6-1. If now the crystal in Fig. 1a is cut along the dotted line which passes through points 3 and 6, and the right half of the crystal removed, we obtain the configuration shown in Fig. 1b. It is seen that a closure failure shown by the dotted arrows between points 3 and 6 now exists. The purpose of what follows is to discuss quantitatively the meaning of these Burgers circuits.

In general, the closure failure $b^k$ associated with a given Burgers circuit may be written as [3]

$$b^k = \oint A^k_K \, dx^K,$$

(1)
where $A^K_\ell$ is defined as the distortion tensor which relates the local coordinates in the initial or undeformed state ($K$) to those in the torn or natural state ($k$) [4]. In the present study, lower case Latin letters will be used to denote the natural state, while upper case Latin letters will be used to denote the initial state. Stoke's theorem may also be used to convert the line integral of Eq. (1) to a surface integral as follows:

$$ b^K = \int_s \partial L A^K_\ell \, dF^{LK} = \int_s \frac{1}{2} [\partial L A^K_\ell - \partial^K A^K_\ell] \, dF^{LK}. $$

Fig. 1. Burgers circuit associated with a) the interior of a perfect crystal b) the surface of a perfect crystal

Still another way to write Eq. (2) is in terms of a surface integral referred to the final state which gives

$$ b^K = \int_s \frac{1}{2} A^K_\ell A^K_m [\partial L A^K_\ell - \partial^K A^K_\ell] \, dF^{lm}. $$

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