"HOW CAN WE DETERMINE THE TWO-BODY t-MATRIX?"

By

J. S. LEVINGER

RENSSELAER POLYTECHNIC INSTITUTE, TROY, NEW YORK, 12181 USA*

We start with an assumed shape for the wavefunction of the ground state of the two-nucleon system. This wavefunction gives us the residue of the t-matrix at the bound-state pole and using the unitary pole approximation (UPA) provides a good extrapolation procedure for other off-shell values of the t-matrix. (E. HARMS has verified the accuracy of the UPA using the anti-bound state for the REID soft-core singlet potential.)

We now face the problem: what bound-state wavefunction should we assume? In the next several years it should be possible to determine the deuteron wavefunction experimentally, by measuring elastic electron-deuteron scattering from polarized deuterons, or equivalently by measuring the polarization of the recoil deuteron. (The wavefunction at large neutron-proton separation should also be checked by using it in the Schrödinger equation to give a local potential which should agree well with that for one pion exchange.) Angular distribution measurements provide a separation of the deuteron form factor into a charge form factor and a magnetic moment form factor. Polarization measurements separate the charge form factor into a monopole form factor $G_0$ and a quadrupole form factor $G_2$. T. BRADY has found that values of $G_2(q)$ at momentum transfers $q$ of order $3 F^{-1}$ are quite sensitive to the percentage of D-state ($p_D$) in the deuteron. At present the lack of knowledge of $p_D$ is the main source of uncertainty in calculation of the energy of the trinucleon.

1. Introduction

We are trying to calculate the properties of the trinucleon (the three-nucleon system, in its bound state $^3$H or $^3$He). The Faddeev equations show us that if we limit ourselves to a non-relativistic three-nucleon problem with only two-nucleon forces, that "all" we need to know are the off-shell values $t(p, k; s)$ of the non-relativistic two-nucleon t-matrix. The momentum of the two-body system is initially $p$, and changes to $k$. The energy of the two-body system (in the center of mass co-ordinate system) is $s$, which in general is neither $p^2$ nor $k^2$. (I use units with $\hbar^2/M = 1$. Also, I am simplifying notation by considering only a central force in a state of specified angular momentum: e.g., the t-matrix for the $1S$ state. I will maintain this over-simplified notation even when discussing tensor forces, for which we need 3 different functions which for the coupled $3S_1 - 3D_1$ deuteron are $t_{00}(p, k; s)$, $t_{02}(p, k; s)$ and $t_{22}(p, k; s)$.)

The momenta $p$ and $k$ have the ranges $0 \leq p < \infty$, and $0 \leq k < \infty$, but we do not need accurate knowledge of $t$ at very high momenta. If we cal-

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culate the properties of the trinucleon at energy $E$, we need values of $s$ in the range $s \leq E$. In this paper I shall neglect the very interesting problems of nucleon-deuteron scattering states above the threshold for deuteron break-up. I thus limit myself to non-positive values of $s$.


Two of the invited speakers here (Mittra and Kharchenko) pioneered the use of a separable approximation to the $t$-matrix. The approximation $t(p, k, s) = g(p) g(k)/D(s)$ reduces by one the dimensionality of the Faddeev equations, thus greatly simplifying their solution. Even with tensor forces, we need to solve only three coupled one-dimensional equations. I wish to emphasize that in using a separable approximation to the $t$-matrix, we are not asserting that the potential is really separable (Levinger, [1]). Of course, if the $t$-matrix were exactly separable, then so is the potential and vice-versa. (Yamaguchi, [2].) But as illustrated below for the Reid singlet soft core potential (Harms, [3]) a local potential can have a $t$-matrix that is separable to a good approximation.

I also wish to emphasize the present phenomenological nature of the theory of the two-nucleon interaction. If we had a good theory, comparable to that for Coulomb forces, we would be able to calculate the off-shell $t$-matrix from first principles, thus answering the question I ask in my title. (Of course we must turn to experiment to determine numerical values of a small number of parameters in the theory, such as the electron charge and the photon rest mass for the case of Coulomb forces.) I believe that nuclear theory is still strongly phenomenological. The one generally accepted statement on the nucleon-nucleon potential is the validity of the one-pion-exchange-potential (OPEP) at reasonably large distances. Even here there is not complete agreement as to what is the range of distance for a given accuracy for a given term in OPEP. (Lomon [4]; Feshbach [5]). Of course, the two parameters in OPEP are determined experimentally, by performing independent experiments.

In the next Section, I outline the "conventional extrapolation procedure" which uses a local potential to extrapolate from on-shell ($s = p^2 = k^2$) to off-shell values of the $t$-matrix. I do not discuss other extrapolation procedures, due to Amado [6]; Baranger [7]; van Dijk [8]; Fuda [9], [10]; Kowalski [11]; HafTel [12] and others. These are based on on-shell values of the $t$-matrix. In Section 3, I present an answer to the preliminary question of my title by using elastic electron-deuteron scattering to determine the deuteron wavefunction, which in turn is used to determine the off-shell values of the $t$-matrix. The crucial experiments of measuring the polarization of the recoil deuteron seem feasible. I hope that the program of Section 3 will in the next several years materialize into a practical procedure for finding the triplet $t$-