Dispersion relations in theories with indefinite metric are discussed. Conditions under which ghost contributions appear are given.

1. Dispersion relations in the theory of fields follow from several basic assumptions. These are: a) Lorentz covariance; b) causality; c) asymptotic condition; d) unitarity, from which e.g. the optical theorem follows; e) and spectrality, which determines e.g. the behaviour of the scattering amplitude in the non-physical region. In the following dispersion relations are analysed in a field theory with indefinite metric, where the conditions a)–d) are fulfilled. The result is that dispersion relations are valid also in this case, but in the non-physical region additional ghost contributions may emerge.

Since in a field theory with indefinite metric not all the conditions are trivial, or even better the unitarity condition is fulfilled only in theories with multipole-type ghosts, we have to formulate these assumptions here more precisely. We restrict ourselves to the treatment of forward scattering and to a model theory of Froissart [1]. It will be seen, however, that these restrictions are not essential.

2. First we quote the main features of Froissart’s model. The model starts with two real scalar fields satisfying the field equations

\[(\Box - \mu^2) A = 0,\]

and commutation relations

\[
[A(x), A(x')] = 0, \quad [A(x), B(x')] = i\delta(x - x'),
\]

\[
[B(x), B(x')] = 2i \lambda^2 \frac{\partial}{\partial \mu^2} \Delta(x - x').
\]
\[ A(x) = A_0(x) = \sum \frac{1}{\sqrt{2\omega}} \left( a(k) e^{ikx} + a^*(k) e^{-ikx} \right), \]

\[ B(x) = B_0(x) + \frac{\lambda^2}{\mu^2} (1 + x_\mu \delta_{\mu}) A_0(x), \quad (3) \]

\[ B_0(x) = \sum \frac{1}{\sqrt{2\omega}} \left( b(k) e^{ikx} + b^*(k) e^{-ikx} \right), \quad k^2 = -\mu^2, \]

\[ [a, a^*] = [b, b^*] = [a, b] = 0, \quad [a(k), b^*(k')] = \delta_{kk'}. \quad (4) \]

Defining \( P_\mu \) as usual by

\[ [P_\mu, A] = i\partial_\mu A, \quad [P_\mu, B] = i\partial_\mu B, \quad (5) \]

one obtains

\[ [P_\mu, B_0] = i\partial_\mu (B_0 + \frac{\lambda^2}{\mu^2} A_0). \quad (6) \]

Let us define the vacuum state as

\[ a|0\rangle = b|0\rangle = 1, \quad (7) \]

and the other states by applying the creation operators \( a^*, b^* \) to it. Thus the state vector space is of the type \(|N_A, N_B\rangle\). From (4) follows

\[ <0, N_B|0, N_B> = <N_A, 0|N_A, 0> = 0, \quad (8) \]

\[ <0, N_B|N'_A, 0> = \delta_{NN'}, \quad (9) \]

but from (6)

\[ (P_\mu - p_\mu)|N_A, N_B\rangle \neq 0, \quad (10) \]

more precisely \( N_B = 1 \) is a dipole ghost state, \(|N_A, N_B\rangle\) is a state with a pole of \( N_B + 1^{\text{th}} \) order. E. g. (10) for the special case of \(|0, p_\mu\rangle\) becomes

\[ (P_\mu - p_\mu)|0, p_\mu\rangle = \frac{\lambda^2}{\mu^2} p_\mu |p_\mu, 0\rangle. \quad (11) \]

The model is obviously relativistic.

3. We are setting up dispersion relations. For this let us consider the interaction of four fields, a meson field \( \varphi \) (with the mass \( m \)), a nucleon field \( \psi (M) \) interacting with each other and with the Froissart fields \( A \) and \( B \). Thus we have two complete sets of state vectors, the in and out states which are of the type

\[ |N_\varphi, N_\psi, N_A, N_B; \text{in}, \text{out}\rangle. \]