THEORY OF ANHARMONIC CRYSTALS
IN PSEUDOHARMONIC APPROXIMATION

III. CRYSTAL WITH WEAK COUPLING

By

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The dependence of the instability temperature on the arbitrary external pressure is investigated for a crystal with weak coupling.

In a previous paper [1] the properties of an anharmonic linear chain under arbitrary external tension were considered in a pseudoharmonic approximation. In this paper I present an additional investigation of the properties of a chain in which the coupling of atoms is weak \[ \lambda = (\pi D/\omega_{0L}) \leq 2 \]. As I established in the earlier paper it is necessary in this case to investigate the properties of the chain in the low temperature limit.

It was shown in [1] that the self-consistent equation which determines the properties of the chain can be written

\[
\lambda x y(x) = \int_0^{\pi/2} d\varphi \sin \varphi \coth \frac{x \sin \varphi}{2\tau},
\]

where the notations are the same as in [1] and

\[
y(x) = \ln \frac{x^2 - \frac{P^*}{6}}{\left(x^2 - \frac{P^*}{3}\right)^2}.
\]

In the low temperature limit the self-consistent equation (1) can be rewritten in the form

\[
\lambda x y(x) = 1 + \frac{\pi^2}{3} \left(\frac{\tau}{x}\right)^2.
\]

The instability temperature can be obtained as a simultaneous solution of Eq. (3) and its derivative [1]:

\[
\lambda \{y(x) + xy'(x)\} = -\frac{2\pi^2}{3} \frac{\tau^2}{x^3}.
\]
The critical temperature can be obtained as a simultaneous solution of Eqs. (3) and (4) and the second derivative of (3):

$$\lambda \{2y'(x) + xy''(x)\} = 2\pi^2 \frac{\tau^2}{x^4}. \tag{5}$$

It is convenient to rewrite Eqs. (3)–(5) in the following form:

$$\lambda = \frac{1 + \frac{\pi^2}{3} \left( \frac{\tau}{\alpha} \right)^2}{xy(x)}, \tag{6}$$

$$\lambda = \frac{1 + \pi^2 \left( \frac{\tau}{\alpha} \right)^2}{x^2 y'(x)}, \tag{7}$$

$$\lambda = \frac{1 + 2\pi^2 \left( \frac{\tau}{\alpha} \right)^2}{x^2 y''(x)} \tag{8}$$

It is easy to see that if \( P^* = 0 \) Eqs. (7) and (8) are incompatible and consequently there can be no critical temperature. The analytical solution of the

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**Fig. 1.** The dependence of the instability temperature \( \tau_s = \Theta_s / \omega_{sL} \) on the dimensionless coupling constant \( \lambda \) of the atoms.