A POINT DYNAMIC MODEL FOR THE CAUSAL INTERPRETATION OF WAVE MECHANICS

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A universal method is found to express the real amplitude $A$ of the complex wave-function $\psi = Ae^{iS/h}$ as the real function of $S$ (action) from the continuity equation of wave mechanics. In this way, quantum potential may be given as a function of $S$. Thus the wave-mechanical eikonal equation contains the function $S$ only. Considering the resulting equation as the wave-mechanical generalization of the Jacobian point dynamical equation, the wave-mechanical generalization of the Newtonian equation of motion might be given. Moreover, as a consequence, it is pointed out that the $\psi \psi^* = A^2$ really means density of particles or the probability of their finding even in this interpretation.

It is known that by means of the usual substitution $\psi = Ae^{iS/h}$ into the time-dependent Schrödinger equation of the complex matter-wave function $\psi$, a coupled pair of two partial differential equations can be derived, both containing real functions $A$ (amplitude) and $S$ (action). One of these is a continuity equation, while the other has a Hamilton—Jacobian feature [1]. It is shown in a preliminary study [3], that function $A$ can always be expressed as a function of $S$, if a proper transformation on the continuity equation is applied. When function $A$, obtained in this way, is substituted into an equation of Jacobi type, this equation conveniently contains only the $S$ action function (§ 1). Encouraged by the critical studies of L. de Broglie and his followers ([4], [5], [6]), an attempt is made in this report to conceive this equation as a wave-mechanical generalization of a Hamilton—Jacobian equation (see [7]), and based upon this concept we try, illegitimately, to maintain in wave-mechanics a classical notion of the mass point and its orbit. This interpretation can be regarded as unitary, since it is sufficient to suppose merely the reality of the particle, and from this supposition the existence of matter-waves can be deduced. It is pointed out that a possibility for the generalization of the classical (Newtonian) equation of motion arises and so, certain conservation

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1 The various interpretations in wave-mechanics are based on this splitting. They have the common well-known aim of interpreting wave-mechanics in the "language" of classical mechanics, giving a generalization of that in this sense [2]. In contrast, an opposite custom is also used, which is based upon the Ehrenfest theorem of mass-centre. This custom has, anyway, certain logically imperfect traits, such as notions of mass and potential energy which are adopted initially from classical mechanics.

laws are modified. Simultaneously, new dynamic parameters are encountered, unobserved (hidden) in classical dynamics (§ 2). As an example, the case of force-free motion in one dimension is treated as well as the wave-mechanical generalization of the inertia law (§ 3). Furthermore, it is proved that in the case of a multitude of incoherent mass-points, $\psi\psi^*$ represents a relative statistical density of them even according to the present interpretation (§ 4). Finally, the point motion is treated before and after the one-dimensioned potential-jump (§ 5).

§ 1. Elimination of the amplitude-function

We may write the time-dependent Schrödinger equation

$$-\frac{\hbar^2}{2m} \Delta \psi + V \psi = -\frac{\hbar}{i} \frac{\partial \psi}{\partial t}$$  \hspace{1cm} (1,1)$$
valid for the complex mass-wave function $\psi(r, t)$ of a point of mass $m$, which is moving in the field of a generalized potential $V = V(r, t)$. Using transformation

$$\psi = A \cdot e^{iS/\hbar},$$  \hspace{1cm} (1,2)
where $A = A(r, t)$ and $S = S(r, t)$ are real functions, Eq. (1,1) can be put into the following two real differential equations:

$$\frac{1}{2m} \text{grad}^2 S - \frac{\hbar^2}{2m} \frac{\partial A}{A} + V + \frac{\partial S}{\partial t} = 0,$$  \hspace{1cm} (1,3)
$$\text{div} \left( \frac{A^2}{m} \text{grad} S \right) + \frac{\partial A^2}{\partial t} = 0.$$  \hspace{1cm} (1,4)

Here (1,3) resembles the Bernoullian equation of dynamics of media and the Hamilton—Jacobian equation of dynamics of points, while (1,4) shows a resemblance to continuity equations of media. In (1,3)

$$U = -\frac{\hbar^2}{2m} \frac{\partial A}{A}$$  \hspace{1cm} (1,5)

is usually qualified as quantum [1] or density-potential [10]. Now we show that $A$ can be expressed from (1,4) as a function of $S$. Instead of $A$, let us introduce the real function $C$ by the transformation

$$A = e^C.$$  \hspace{1cm} (1,6)
