THE HALL EFFECTS ON HYDROMAGNETIC FLOW OVER A PERMEABLE BED

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Hydromagnetic forced convection in a parallel plate channel bounded by a rigid insulated plate and a permeable bed and permeated by a uniform transverse magnetic field has been considered taking Hall effects into account. Solutions for the flow above the bed, Zone 1 and that below the bed, Zone 2 are obtained using the matching conditions at the interface and also suitable boundary conditions at the bed. The primary flow, secondary flow, induced magnetic field components and the temperature distribution are found. The shear stresses and the Nusselt number at the bed and at the plate are calculated. Hall currents are found to exert a profound influence on the flow and heat transfer characteristics.

1. Introduction

The importance of flows through and past porous media in technology, geohydrology, petroleum industry and geophysics is indisputable. The flow through porous media is usually determined using Darcy's empirical formula. Beavers and Joseph [1], Saffmann [2], Taylor [3] and Rajasekhar [4] have investigated flow past horizontal porous beds. The temperature distribution for a Poiseuille was examined by Vidyaniidhi, Sittapati and Narayana [5] and for plane couette flow in the presence of buoyancy forces was studied by Rudraiah and Veerabhadraiah [6]. The Hartmann flow past a permeable bed in the presence of a transverse magnetic field was investigated by Rudraiah, Ramaiyah and Rajasekhar [7] to illustrate the experimental work of Wallace, Pierce and Swayer [8]. The aim of this paper is to take into consideration the Hall effects and study these effects on the flow and heat transfer characteristics. Such a study will be of some use in the problem of cooling nuclear reactors where very strong magnetic fields are used and also in the utilization of the enormous power beneath the Earth's crust in the geothermal fields, which are clearly a problem of flow past a porous medium with the Earth's surface as a naturally permeable bed.

Here we consider the flow of an electrically conducting liquid through a parallel plate channel $z' = \pm 1$ bounded below by a permeable bed. We suppose that strong uniform magnetic field $H_0$ acts along the $z'$-axis. A uniform pressure gradient is maintained in the longitudinal direction in both the channels and the permeable material. In Zone 1 the flow is laminar and

is governed by magnetohydrodynamic equations (with Hall effects included) while the flow in the Zone 2 is determined by the modified Darcy's law. We use the slip velocity boundary condition [1] at the permeable bed. We further assume that the upper plate is at temperature $T = T_1$ while on the permeable bed we adopt the thermal slip boundary condition considered by Rudraiah and Veerabhadraiah [6] and also independently by Vidyanidhi and Narayana [9]. It is well known that the introduction of Hall effects produces a cross flow [10]. The present investigation thus gives a complete picture of the flow and heat transfer characteristics when the Hall parameter is present.

2. Mathematical formulation

The physical model consists of two Zones; in one Zone, from the impermeable upper rigid plate up to the permeable bed, the flow called the modified Hartmann flow (due to Hall effects) is governed by magnetohydrodynamic equations and in the other Zone below the permeable bed, the flow is determined by the modified Darcy flow. In the following, we shall call the former Zone 1 and the latter Zone 2. The basic equations and the corresponding boundary conditions are set up for Zone 1 and 2, respectively. Solving these equations, the solutions are matched at the interface to get uniformly valid solutions throughout the region of flow.

The basic equations [10] are

\[ \nabla' \cdot \vec{V}' = 0, \]

\[ (\vec{V}' \cdot \nabla') \vec{V}' = -\frac{1}{\varepsilon} \nabla' p' + \nu \nabla'^2 \vec{V}' + \frac{\mu_e}{\varepsilon} \vec{J}' \times \vec{H}_0. \]  \hspace{1cm} (1)

Maxwell's equations are

\[ \nabla' \times \vec{E}' = 0, \quad \nabla' \times \vec{H}' = \vec{J}'. \]

along with Ohm's law including Hall effects given by

\[ \vec{J}' + \frac{\omega \tau}{H_0} \vec{J}' \times \vec{H}' = \sigma_e [\vec{E}' + \mu_e \vec{V}' \times \vec{H}'], \]  \hspace{1cm} (2)

where $\vec{J}'$, $\omega$, $\tau$, $\mu_e$, $\sigma_e$ denote the current density, electron Larmor frequency, electron collision time, magnetic permeability and the electrical conductivity, respectively. In writing the Eq. (2), the ion slip effects arising out of imperfect coupling between ions and neutrals as well as electron pressure gradient are neglected.

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